What's in a task?

Generating mathematically rich activity

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What's in a task? - generating mathematically rich activity.

'Doing mathematics' always integrates thinking mathematically, using mathematical knowledge, and using mathematical enquiry methods. Rich mathematical activity suggests ***active*** student learning. It requires teachers to select purposeful tasks and plan questioning that encourages all learners to show what they know and to extend that learning. Different tasks can be used to cultivate different types of skills and thinking yet all tasks should be about 'doing mathematics'. This approach to teaching addresses the assessment objectives for both of the linked-pair pilot GCSEs. Teaching practices can be similar across both GCSEs if it generates rich mathematical activity.

The potential 'richness' of a task is evident from its context, its complexity, its novelty or its requirement for analysis, synthesis or evaluation. Rich contextual tasks can be sourced readily, but it's what the teacher does with the task that really counts. Rich mathematical activity can also be generated in complex mathematical contexts, and from simple mathematical questions, so that engaging students will prepare them for both GCSE qualifications which aim for "rich experience of mathematics, enabling them to understand its importance in analysing problems related to both the real world and to mathematics itself."

Teaching that focuses on mathematical thinking and developing new mathematical understandings can: (a) offer all aspects of the national curriculum, (b) prepare students for both qualifications, and (c) maintain engagement with mathematical ideas.

<u>The challenges in generating rich mathematical activity are:</u>

- Being mathematical and sustaining the focus on mathematics
- Believing that all students can engage in mathematical enquiry and are learning
- Balancing the freedom, discussion and frustrations that go with rich activity with the need to support students to understand new ideas.

This document includes three tasks that illustrate what we mean

Task 1 Routine mathematical task (on page 7)

Task 2 Non-mathematical context (on page 8)

Task 3 Mathematical context (on page 9)

The tasks involve:

- Diversity in Tasks 1, 2 & 3
- Depth and connectedness in Tasks 1, 2 & 3
- Prior knowledge for Tasks 1, 2 & 3
- Teaching & Awareness of new mathematical ideas Tasks 1, 2 & 3

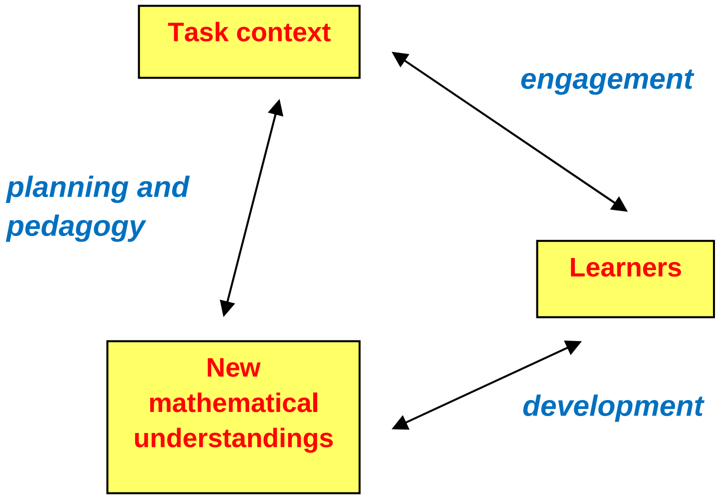
Generating rich mathematical activity

Richness is created by the level of questioning, diversity of approaches, and exploiting the potential depth and connectedness of mathematics, whatever the starting point.

Questioning – for prompting, encouraging, challenging, and clarifying – is the key practice of the teacher wishing to generate rich mathematical activity. In other words, **questioning to provoke learning rather than to ascertain what has been learned**. Questions need to be planned in advance to ensure that *mathematical* questions are asked, otherwise the context and general problem solving become the focus rather than mathematical progress.

There have to be strong and obvious connections between:

- the task context and new mathematical understandings; links are more likely to be made if planned for and if the new knowledge is necessary
- the context and the way learners are expected to engage with it; how will they bring their existing knowledge to the task – mathematical knowledge and 'outside' knowledge?
- access to the new potential mathematical understandings; do these develop from what they already know and can do?



Tasks 1, 2 & 3 have the potential to generate rich mathematical activity because:

- they cannot be completed using only low level mathematical activity,
- they draw on prior knowledge,
- there is no single 'right way' to do these – many ideas and 'answers' can be generated and evaluated,
- teaching makes a difference to what is learnt, and
- they have the potential to raise students' awareness of new mathematical ideas.

Questions that generate rich mathematical activity can be defined in the following categories:

Questioning the context:
What is meant by …?
What is another example or case of …?
What are the variables?
What if …?

Problem-solving questions:
What objective is required? Is there sufficient information? Is there superfluous information? How can the scenario be represented? How reasonable is my solution?

Mathematisation within the task:
What patterns are apparent? How can the problem be modelled? What mathematics skills or knowledge could be used to determine a solution? What degree of accuracy is required? What does my solution mean in terms of the original context?

Mathematisation beyond the task:
How can my solution be generalised?
What else can I use the generalisation for?
What other questions can be explored using the same or similar methods?
How do I understand mathematics

Rich mathematical tasks

In the literature there are many and varied definitions of 'rich tasks'. Two particularly useful definitions have been produced in *Engaging mathematics for all learners* (a document produced by QCDA) and from Jenny Piggott, ex-director of NRICH.

Engaging mathematics for all learners from QCDA says

"A rich mathematical task:

- should engage everyone's interest from the start
- allows further challenges and is extendable
- invites learners to make decisions about how to tackle the activity and what mathematics to use
- involves learners in speculating, hypothesis making and testing, proving or explaining, reflecting, interpreting
- promotes discussion and communication
- encourages originality and invention
- may contain an element of surprise
- is enjoyable
- allows learners to develop new mathematical understandings."

The description used by Jenny Piggott incorporates those used by others, and emphasises mathematical learning, i.e. the purpose of engagement in mathematical tasks of any kind rather than only problem-solving processes.

She describes rich tasks as having these characteristics:

- are accessible to a wide range of learners
- might be set in contexts which draw the learner into the mathematics
- are accessible and offer opportunities for initial success, challenging learners to think for themselves
- offer different levels of challenge
- allow for learners to pose their own problems
- allow for different methods and different responses
- offer opportunities to identify elegant or efficient solutions
- have the potential to broaden students' skills and/or deepen and broaden mathematical content knowledge
- encourage creativity and imaginative application of knowledge
- have the potential for revealing patterns or lead to generalisations or unexpected results
- have the potential to reveal underlying principles or make connections between areas of mathematics
- encourage collaboration and discussion
- encourage learners to develop confidence and independence as well as to become critical thinkers.

Where do you get suitable tasks?

www.nrich.maths.org (linked to national curriculum stages and degrees of difficulty)
On this website you will find thousands of free mathematics enrichment materials (problems, articles and games) for teachers and learners from ages 5 to 19 years. All the resources are designed to develop subject knowledge, problem-solving and mathematical thinking skills. The website is updated with new material on the first day of every month.

http://www.atm.org.uk/resources/ (tasks that have been used and / or developed by practicing teachers)
The Association of Teachers of Mathematics believes in providing teachers with the resources to help them develop their mathematics teaching in creative and broad-thinking ways. The ultimate aim being to develop a creative and thinking approach in mathematics learners.
ATM publishes a wide range of resources and materials. They all have one thing in common: they are written by teachers for teachers and for learners.

http://www1.curriculum.edu.au/maths300 (mathematical and realistic contexts and access to software)
Maths300 is an exciting web-based project which aims to support teachers in the delivery of excellent mathematics education. The aim is to resource members with extensive notes for, at least, the best 300 maths lessons (K-12). Lessons have been successfully trialled and are presented in the illustrative MCTP^ style. About one third of the lessons are supported by specially written, down loadable software. In addition, lessons are supported by Investigation Sheets (with answers), game boards where relevant, and a regular newsletter.

^MCTP – Mathematics Curriculum and Teaching Program: one of the most successful professional development programs to improve maths in grades K – 10 in Australia.

And don't forget that any mathematical task can generate rich activity with good questions.

Qualifications

The linked-pair specifications are very similar to one another, so we have lined them up together to show that the job of teaching is similar for both:

Applications of mathematics *GCSE* must enable learners to	**Methods in mathematics *GCSE* must enable learners to**
• develop knowledge, skills and understanding of mathematical and statistical methods, techniques and concepts • select and apply appropriate mathematics and statistics in everyday situations and contexts from the real-world • use mathematics to represent, analyse and interpret financial information • understand and use the statistical problem solving cycle • acquire and use strategies for problem solving and modelling in context, understanding that models may need refining and that there may be more than one way to solve a problem • interpret mathematical results and draw and justify conclusions that are relevant to the context • communicate mathematical information in a variety of forms.	• develop knowledge, skills and understanding of mathematical methods, techniques and concepts • make connections between different areas of mathematics • select and apply mathematical methods in mathematical contexts • reason mathematically, construct arguments and simple proofs, and make logical deductions and inferences • develop and refine strategies for solving a range of mathematical problems • communicate mathematical information in a variety of forms.

Are they learning?

In recent years we have become accustomed to the requirement that every lesson should have objectives and that learning be assessed continuously through student feedback to inform our teaching. The skills, dispositions and habits of mind necessary to be able to 'do mathematics' in the full sense are not so easily labelled, targeted, and assessed as individual concepts and methods might be. To pass GCSE, students need to develop their abilities to:

- Recall and use their knowledge of the prescribed content
- Select and apply mathematical methods in a range of contexts
- Interpret and analyse problems and generate strategies to solve them.

These are the requirements of both courses in the linked-pair, so teaching methods have to be similar in both contexts. Research tells us that these habits of mind all need multiple varied experiences over time to develop. Learning to select and apply methods has to include making unhelpful decisions and understanding how to ‘spot’ this and rectify it. Learning to interpret and analyse problems has to include many varied problems, including some intractable ones. Generating strategies for yourself is not the same as applying learnt strategies, but the experience of applying given strategies helps you understand how and when they work. Research evidence about this can be found on the Nuffield Foundation website (http://www.nuffieldroundation.org/filelibrary/p7.pdf).

How do teachers know that their students are learning? By listening to what they say and their own ideas about tackling tasks; by providing help and support in the form of modelling good questioning and mathematical enquiry; by providing multiple experiences for the development of the habits of 'doing mathematics' in all its forms.

Learning objectives have to be medium term and about developing habits of mind as well as being short term and about the mathematics within and beyond the task.

Task 1:

Write down the next two terms in each of these arithmetic sequences:	
1, 3, 5, 7 …	1, -1, -3, -5, …
1, 4.5, 8, 11.5 …	8, 6, 4, 2, …
46, 48, 50, 52 …	5, 10, 15, 20 …

For Task 1, the teacher could ask questions relating to:

1. **Questioning the context**: *What do these have in common and how do they differ? What can we say in general about them?*

2. **Problem solving questions:** *What questions can be posed about these? Have I met anything like this before? What is my plan of action to explore these further?*

3. **Mathematisation within the task:** *What are the variables and how do they relate? Can I predict the 'next two terms' of any sequence which is like these in some way? Can I predict other terms? What does my solution mean in terms of the dimensions of the original context? If I sorted them, what classifications could I use?*

4. **Mathematisation beyond the task:** *What other sequences can be predicted? What mathematics can I do now that I could not do before? What mathematics do I understand differently as a result of doing this task? What is worth remembering from this?*

With suitable questioning, students could engage with the following processes in order to understand generalities about arithmetic sequences.

Sort	Look for special cases	Look for same/different
Compare	Represent	Identify variables
Classify	Generalise	Change parameters
Identify and describe patterns	Devise or use notation	Extend domain of exploration
Exemplify	Predict	Deduce

This task provides an opportunity for students to demonstrate their existing mental arithmetic skills, to make a shift from additive to multiplicative thinking, and to express generalities algebraically. Teachers need to consider contexts and learning experiences that enable students to see the benefit in this shift of thinking, i.e. from thinking about the arithmetic of sequences to thinking about the algebra through controlling variables.

Task 2:

A chocolate manufacturer wishes to wrap individual ‘Chockynutclusters’ in red foil for Valentine’s Day gifts. Foil, which is more expensive than chocolate, comes in rectangular sheets 1m by 1.5 m. Each chocolate is roughly spherical in shape and the current radius measurement is (3 ± 0.25) cm. This measurement can be varied by up to 15% by volume. Should the size be changed? How should the wrappings be cut out most economically?

For Task 2, the teacher could ask questions relating to:

1. **Questioning the context:** *What is another case of wrapping we could use to compare, to visualize, to learn from …? If we change the size, does it matter how many bites each chocolate is? What does ‘roughly spherical’ mean?*

2. **Problem solving questions:** *What objectives are required? Is there sufficient information? Is there superfluous information? Are there any sub-goals worth pursuing first? Have I met anything like this before? What is my plan of action? How reasonable is my solution?*

3. **Mathematisation within the task:** *What are the variables and how do they relate? How can the problem be modelled? What maths skills or knowledge could be used to determine a solution? What do I need to know or find out? What degree of accuracy is required? What does my solution mean in terms of the dimensions of the original context?*

4. **Mathematisation beyond the task:** *What other situations could be solved using the same methods? What mathematics can I do now that I could not do before? What mathematics do I understand differently as a result of doing this task? What is worth remembering from this?*

Part of the role of teachers in generating rich mathematical activity is modelling and actively referring to mathematical processes.

Task 2 offered opportunities to:

Simplify	Estimate	Exemplify
Represent	Visualise	Generalise
Devise or use notation	Model	Tabulate
Use formulae	Relate	Infer

These methods of mathematical enquiry can be employed to learn about percentage change, ratios of similar 3-D shapes, and circular and spherical mensuration. In this task students can be given new formulae as tools for this problem, when they need them, rather than as a prerequisite. Understanding these can start through use, and other aspects, such as dimensionality, can come later.

Task 3:

Determine some quadratic functions whose roots are 2 units apart. Compare methods with other students and decide what is the quickest, most efficient, most productive method for finding as many as possible. Why does your method work?

For Task 3, the teacher could ask questions relating to:

1. **Questioning the context:** *What is a quadratic function? What are 'roots'? What else do I know like this?*

2. **Problem-solving questions:** *What are the objectives? Are there any sub-goals worth pursuing first? What do I know already and what do I need to find out? Have I met anything like this before? What is my plan of action? How reasonable are my solutions?*

3. **Mathematisation within the task:** *What representations can I use to get a sense of what this is about? What maths skills or knowledge could be used to determine a solution? Do I need to review earlier knowledge? Can ICT help me think about this? Would it help to start with some examples and see what happens? What do my solutions mean in terms of the original context?*

4. **Mathematisation beyond the task:** *How can my solution be generalized: to other intervals, to other functions? What other questions could be explored using the same methods? What mathematics can I do now that I could not do before? What mathematics do I understand differently as a result of doing this task? What is worth remembering from this?*

Task 3 offered opportunities to:

Represent	Devise or use notation	Exemplify
Identify and describe patterns	Look for special cases	Generalise
Compare	Classify	Predict
Deduce	Switch representations	Extend domain of exploration

These methods of enquiry can be employed to learn about quadratics, relations between roots and coefficients and the shape of graphs, formulaic approaches to finding roots, what 'solving a quadratic' means, and the transformation of graphs.

We are assuming that, if students know what a quadratic function is, they will be familiar with its representation as $y = ax^2 + bx + c$ and *not* with $y = (x-a)(x-b)$. An important stage in thinking about non-routine situations is 'what do I need to know?' and, in mathematical contexts, 'what mathematical tools can I use to get a handle on this'. But there is little point in waiting for students to *discover* how to find roots for themselves. Let's look at some ways forward that would be available for students who have regularly engaged in rich mathematical activity.

Decide on a representation, graphical (using graph plotter) or algebraic; see if they can construct an example that gives them roots that are a distance of '2' units apart. Graphically this could be by transforming, changing parameters in an expression until the roots 'look' right and then seeing what they can vary and keep the roots 2 units apart. This exploits a key mathematical question 'what varies and what stays the same?' Algebraically, they might work backwards from two numbers, say 5 and 7 ‘must have come from (x-5)(x-7)’ and, by multiplying brackets, proceed to an algebraic expression of a familiar form.

Students who do not regularly work in these ways may need some more specific starting points, but this does not mean they also need a structured sequence of steps to go through. For example, a teacher could start by saying that y = (x-a)(x-b) is the formula for a quadratic with roots at *a* and *b*. This gives immediate access to some example generation and generalisation – it almost trivialises the problem but allows for discussion about algebraic representation, comparing this to the ‘usual’ representation so you can see how the roots and coefficients are algebraically related. The teacher might then ask for a *generalisation* in terms of what these functions look like. Another approach might be to pose this question after students have been introduced to graphing software, but then the software may become the context, rather than the quadratic functions.

As with Task 2, it matters what happens next. Are the ‘then what?’ and ‘so what?’ tasks about the context (use of graph plotter or quadratic functions) or general skills (I persisted; I used my prior knowledge) or mathematics (for example, can the distance between the roots be zero? What coefficients give no real roots?). When mathematics is the context it is easier to focus on the development of new understandings. The formula for finding roots makes sense when it arises after this kind of exploration.

Diversity:

Rich mathematical activity is accessible for all students in that the tasks selected by the teacher ideally have multiple entry and exit points, and provision for using a variety of methods, some more sophisticated than others. The nature of the questions used by the teacher will increase the success of all students in making mathematical progress while doing the task.

To this end, the teacher needs to determine the *minimum* outcome and the *core* elements of the activity and think about how all learners can achieve these, including the nature and degree of scaffolding that will be provided.

For example, the *minimum outcomes* for the three tasks described here could be:

Task 1: Students being able to predict later terms OR Students being able to construct formulae for terms and sums of arithmetic progressions with any initial value and common differences.

Task 2: Students deconstructing the wrappers of actual spherical chocolates (e.g. Ferrero Rocher) and measuring the dimensions of the wrapper and the circumference of the chocolate, and comparing with the dimensions of the chocolates in the task, OR Students being able to manipulate error terms appropriately when changing from length to volume.

Task 3: Students draw different parabolas whose roots are 2 units apart and list their equations OR Students use graphical software to investigate families of quadratic functions to find some whose roots are 2 units apart OR Students work from algebraic representations.

The minimum outcomes may require differing degrees of scaffolding by the teacher in order for them to be met be all students. Furthermore, these minimum outcomes may vary according to the age and prior achievement and experience of the students.

Depth and connectedness:

When the task is ‘finished’ what happens next? We think of this as ‘then what’ and ‘so what?’ You can stay with the context, the problem-solving strategies, or the mathematics.

For rich mathematical activity to be generated, the ‘then what?’ needs to be about new mathematical ideas (see www.atm.org.uk/journal/archive/mt182.html for more)

- In Task 1 we can ask: *Are we able to predict what the 20th term will be? How can we generalise each sequence to determine any term?*

- In Task 2 we can ask: *What can we do next that also involves spherical measurements, or relations between changes in length and changes in area and volume, or spreadsheet representations of covariation that is not linear?*

- In Task 3 we can ask: *What can we do next that involves roots of polynomials, or relations between algebraic and graphical representations, or quadratics that do not intersect with the x-axis?*

- In Task 3, questions and ‘then whats’ about context and problem solving strategies will also be about methods of mathematical thinking and enquiry, but in Task 2 the distinction between mathematical, generic and contextual ‘then whats’ is crucial to stop the task rolling away into the world of chocolate.

Prior knowledge:

- Task 1 draws on learners' prior experiences of number patterns and their knowledge that the sequence is generated by the application of a consistent 'rule' such as *add 2 to the previous term.* Fundamentally this task also is reliant upon proficiency with computation, particularly mental arithmetic, because those methods provide the structure for expressing generalities in algebra.

- Task 2 draws on learners' prior knowledge of wrappings so they have some situational knowledge to bring to the task which can be revisited throughout, e.g. *How are those chocolates wrapped? How much overlap is required? How many bites are necessary?*

- Task 3 draws on learners' previous experience of related ideas. These will vary according to previous teaching, experience and preferences with graphs, algebra, ICT, but there are several different starting points so long as there is some minimal knowledge and appropriate tools.

Teaching & Awareness of new mathematical ideas:

For mathematics to happen, students need to have access to, and understand the power of mathematical ideas such as: representations (e.g. the relation between graphs and depiction of roots); relations (e.g. between radius and circumference; between coefficients and roots); formulae (e.g. of volume of sphere); covariation (e.g. how volume varies when radius varies; variation (e.g. how an arithmetic sequence is generated and generalized); how transformations of a function can preserve or change distance between roots).

Teachers have to decide how and when to introduce these ideas.

- Task 1 can be reduced to determining constant addition or subtraction thereby generating the next two terms, or the task can be used to make generalisations about each sequence which may then be used to predict the n^{th} term. The latter approach provides the opportunity for students to make a shift from additive to multiplicative thinking, and teachers need to consider contexts and learning experiences that enable students to see the benefit in this shift of thinking. Two of the sequences have a common difference of 2, and two of -2, so students can be asked to generate more sequences with these common differences and see the effect of having different starting numbers. Generating sequences which have the same starting numbers and different common differences can then be done to see the effects of the common differences. In this way, the formulae for finding terms and sums of arithmetic progressions can be a generalisation task for students.

- Task 2 can be reduced to a sequence of applications of methods to generate a string of numerical information, or the task can be a completely open exploration in which an engineering approach of trial and adaptation is used. *Neither* of these is mathematically rich because they do not offer the opportunity to meet, use, connect or devise new mathematical understandings. Teachers need to direct the learning experiences towards ideas such as how changes in length affect changes in area and volume, for example a 15% variation in volume relates to a 4.8% change in radius.

- Task 3 *can be reduced to a sequence of exercises, or application of methods which the teacher has just introduced, or presented as an open exploration so long as appropriate tools are available in the group and the room. The former approaches do not offer the opportunity to experience how mathematical enquiry can lead to new understandings through conjecture and insight. Teachers need to direct the learning experiences towards ideas such as how the distance between the roots can be used to devise a solution formula for quadratics; how expressing a polynomial as a product of factors opens new lines of enquiry.

Questioning:

Questions can be about context, problem solving, mathematics within the task and mathematics beyond the task. There are many questions that work in most mathematical contexts and there are several collections of these. We direct you to four sources:

1. An excerpt from 'Ways of Working with Maths Plus' for Queensland Investigations Edition © Oxford University Press (2009). http://www.oup.com.au/learning_exchange/primary/open-ended_tasks_and_investigations_in_the_mathematics_classroom

Note that the focus is on being mathematical.

Questions that focus students' thinking in a general direction

- What does this remind you of?
- How many ways can you find to…?
- How could you sort these…?

Questions that assist students to focus on particular strategies and to see patterns or relationships

- What is the same?
- What is different?
- How do you know?
- Can you find more examples?
- What are some things you could try?
- Can you see a pattern?

Questions that require students to explain what they are doing

- What have you discovered?
- Explain to me what you are thinking.
- How did you find that out?
- Why did you do it that way?
- Why do you think you are correct?

Reflective questions to draw together the efforts of all students

- Who has the same solution as this one?
- Who has a different solution?
- Have we found all the possibilities?

- Do you think we have found the best solution?

2. An excerpt from ‘Questions and Prompts for Mathematics Thinking’ (Anne Watson & John Mason), available from © www.atm.org.uk. This publication focuses on questions which encourage exploration of mathematical ideas.

Exemplifying Specialising	Completing, Deleting, Correcting	Comparing, Sorting, Organising
Give me one or more examples of ... Describe, Demonstrate, Tell, Show, Choose, Draw, Find, Locate, an example of ... Is ... an example of ...? What makes ...an example? Find a counter-example of ... Are there any special examples of ...?	What must be {added / removed / altered} in order to {allow / ensure / contradict} ...? What can be {added / removed / altered} without affecting ... ? Tell me what is wrong with ... What needs to be changed so that ...?	What is the same and different about ...? Sort or organise the following according to ... Is it or is it not ...?

Changing, Varying, Reversing, Altering	Generalising Conjecturing	Explaining, Justifying, Verifying, Convincing, Refuting
Alter an aspect of something to see effect. What if ...? If this is the answer to a similar question, what was the question? Do .. in two (or more) ways. What is quickest, easiest, ...? Change ... in response to imposed constraints.	Of what is this a special case? What happens in general? Is it always, sometimes, never ...? Describe all possible ... as succinctly as you can. What can change and what has to stay the same so that ... is still true?	Explain why ..., Give a reason ... (using or not using ...) How can we be sure that ...? Tell me what is wrong with ... Is it ever false that ...? (always true that...?) ... How is ... used in ...? Explain role or use of ... Convince me that ...

3. You can find more questions and tasks in ‘Thinkers’ (Chris Bills, Liz Bills, Anne Watson & John Mason) available from www.atm.org.uk. This book contains 16 basic question types with lots of examples of their use across the mathematics curriculum. For example, ‘give an example of …’ tasks can often trigger rich mathematical activity:

- *Give an example of a set of numbers whose mean is 5*
- *Construct a spinner for which the probability of a 2 is twice the probability of a 4*
- *Give an example of a right-angled triangle whose hypotenuse is 5*
- *Give an example of a point (x,y) such that: 3x+4y=32;*
- *Give an example of a way of representing a particular set of data*
- *Give an example of a unit fraction which is the sum of two unit fractions*
- *Give an example of a pair of points whose mid-point is (2,3)*
- *Give an example of an event whose probability is 1/6*
- *Give an example of an angle whose sine is 0.5*
- *Give an example of average speeds and times which could be used to travel 120 miles?*

- *Give an example of a point which is a distance three from (4,5)*

4. Boaler and Brodie use the following categorisation of the purposes of questioning, but you have to supply your own actual questions. Their categories focus on generating discussion, sharing ideas, and developing mathematical talk.

Purpose	**When used**	**Examples**
Gathering information	Checking for a method, leading students through a method. Wanting a direct answer, usually wrong or right. Rehearsing known facts/procedures. Enabling students to state facts/procedures	What is the value of x in this equation? How would you plot that point?
Inserting terminology	Once ideas are under discussion, enabling correct mathematical language to be used to talk about them	What is this called in mathematics? How would we write this correctly mathematically?
Probing	Getting students to explain their thinking. Enabling students to elaborate their thinking for their own benefit and for the class	How did you get 10? Can you explain your idea?
Exploring mathematical meanings, relationships	Pointing to underlying mathematical relationships and meanings. Make links between mathematical ideas	Where is this x on the diagram? What does probability mean?
Linking & Applying	Pointing to relationships among mathematical ideas and mathematics and other areas of study/life	In what other situations could you apply this? Where else have we used this?
Extending thinking	Extending the situation under discussion, to where similar ideas may be used	Would this work with other numbers?
Orienting / Focusing	Helping students to focus on key elements or aspects of the situation in order to enable problem-solving	What is the problem asking you? What is important about this?
Generating Discussion	Enabling other members of class to contribute, comment on ideas under discussion	Is there another opinion about this? What did you say, Justin?
Establishing context	Talking about issues outside of math in order to enable links to be made with mathematics at later point	What is the lottery? How old do you have to be to play the lottery?