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PARAMETERS FOR PRACTICE AND RESEARCH IN TASK DESIGN IN MATHEMATICS EDUCATION

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ICMI Study 22 brought together a wide range of current thinking about task design from practice and research perspectives. From contributions to that study I have selected ideas, categorisations and examples that could frame future thinking about task design. These come from a range of theoretical backgrounds, each of which describes design principles in different ways. Rather than using the language of principles, therefore, I use the term 'parameters' to suggest that any scholarly work on task design needs to ensure that each idea is included in the structure of such work, and has an associated commitment or range of commitments. In this way, task designers and researchers from different theoretical traditions can interact at the level of practice.

INTRODUCTION

In July 2013 ICMI Study 22 on Task Design held a conference at which 80 international scholars came together to share and synthesise their ideas. Following the conference, teams of authors selected by ICMI prepared comparisons and summaries of those ideas which, it is hoped, can provide a baseline and springboard for future research and practice. In this paper I have selected some of the key ideas and categorisations from this process. These arose from a range of theoretical and practical backgrounds; coordinating the research associated with them for the final publication of the Study has generated some useful insights. I claim that some of those insights can be encapsulated as parameters for future work. I use the word 'parameters' to mean that subsequent publications on mathematics task design could ensure that each is populated with a theory-specific commitment or range of commitments. The parameters in this paper are not newly stated here, but my aim has been to bring them together in one place. For example, a constructionist approach to task design might prioritise attention to tool use and purposeful construction, but it could be argued that *all* task design benefits from being able to answer the kinds of questions that arise from considering tool and purpose, even if these are not the main considerations.

In this paper I draw heavily on the work of the working groups and IPC for ICMI study 22[1]. They did the initial work of synthesising disparate ideas, but I take full responsibility for presenting a contestable conjecture about application to *all* task design methods to TSG36 at ICME-13.

GRAIN SIZE

One of the working groups for the study discussed the range of theoretical frameworks for task design. An insight from this work was that comparisons between frameworks cannot be made without first considering the grain size of their focus (Kieran, Doorman and Ohtani, 2015). *Grand*

[1] Throughout this paper I refer mainly to the places where these ideas have been synthesised, rather than the myriad authors whose ideas have contributed. This is because of length restrictions, and no disrespect is intended.

theories tend to start with general considerations about the meaning and processes of learning inside and outside educational settings. These might include developmental, cognitive, sociocultural and neuroscience theories[2]. Another kind of grand framework is the structure of mathematical concepts, their interrelationships and interdependencies; it is not enough to assume without question that the experience of learners during a task or sequence of tasks should follow historical genesis, or a particular curriculum, or an axiomatic argument, or a particular cognitive progression.

Intermediate frames are theoretical structures that can guide practice across a variety of areas of mathematics. They each depend on a grand frame, to describe and explain learning, and present complex interactions between task, teacher, teaching methods, educational environment, mathematical knowledge and learning in some kind of structure. The purposes and implications for task design are then understood within the total structure of practice. Some approaches to task design are situated explicitly within particular intermediate frames, and the associated design practices would systematically attend to the appropriate parameters. Table 1 illustrates this in Conceptual Change Theory (CCT). In CCT, learners first express their current understanding, and become aware of limiting assumptions through comparing different ideas. The task generates a need, and the means, for conceptual restructuring. In table 1, designers have created an experience in recognising density of number (Van Dooren, Vamvakoussi, & Verschaffel, 2013).

Design principle	**Task**
Take students' prior knowledge and potential initial understandings into consideration	Draw, describe, your ideas of number line, and of the set of all numbers
Facilitate students' awareness of their background assumptions	Compare different ideas and see what can and cannot be included in your own description
Use models and external representations, know their power and their limitations.	Using a rubber band as a number line, see how there can always be numbers between any marks you put on the line; compare this to using a ruler as number line
Foster analogical reasoning that supports conceptual restructuring.	If you were on the rubber band, standing at the point 2.3, what points would be next to you?

Table 1: Principles and task using Conceptual Change Theory (adapted from Kieran et al. 2015)

Communities of practice can develop around intermediate theories, in which case what I am calling 'parameters' are in effect the discourse, habits of mind, and practices of a community. The intermediate frames explored in the Study are those with associated literature and cultures.

The smallest grain size identified for theoretical frames is *domain specific* or *local*. These frames address particular aspects of the intermediate frames, such as proportional reasoning, group work, use of specific tools and so on, and are applied to tasks and pedagogies that have limited and tightly focused purposes. These are more than the application of an intermediate frame to specific lessons as demonstrated in Table 1. For example, if the pedagogic aim is to encourage exploration of

[2] I added neuroscience to the range of possible theories.

mathematics in a heterogenous group of learners, appropriate characteristics of task design might include accessibility for all learners, and extendability, and some elements of openness (Sullivan, Knott and Yang 2015). However, this design frame would not be appropriate for all mathematical work. From consideration of grain size, the following task design parameters can be identified:

- a general theory of learning
- a commitment to a general structure of mathematical concepts
- connections between task, teacher, teaching methods, educational environment, mathematical knowledge and learning
- a domain or local focus

These commitments would depend, to some extent, on a view of the purpose of education more broadly, and the role of mathematics within education. They also depend on local conditions such as curriculum and assessment regimes and educational aims for specific groups of learners.

PURPOSE OF A TASK

Task purposes depend on the educational aims of mathematics, and there is now a global orthodox view that these should include procedural and conceptual understanding and fluency, mathematical reasoning and enquiry, problem solving capabilities, and a positive disposition towards the subject. I am taking these to be given, but of course not all tasks can serve all aims. Some jurisdictions, some intermediate theories, and some teachers might have additional aims such as: the need to talk, write and listen to mathematics; using reasoning for different purposes, e.g. to conjecture, persuade and prove; using mathematical feedback, such as self-correcting, appreciating comments from others; seeing mathematics as part of citizenship, to understand the world and to relate mathematical work to other human values (Watson and Thompson 2015). Beyond mathematical knowledge and competence in mathematical modes of enquiry, tasks could promote teamwork, discussion, literacy, self-confidence and other generic characteristics.

Tasks are the means by which teachers prompt learners' mathematical activity, so rather than relating the task directly to educational aims, it makes sense to think of purpose in terms of providing experiences through which learners can develop desirable knowledge of, habits of mind and dispositions towards mathematics. Task purpose is therefore a design parameter that needs to be well defined. Firstly, the task might concerned with concepts, principles, skills, and/or problem-solving strategies; secondly, the mathematical ideas used or developed in the task might be new for learners or not. Meeting a concept for the first time is different from applying, restructuring, expressing and building connections with a concept which may be already partly familiar. Shuard and Rothery (1984, pp. 5-6) make further important distinctions between whether the purpose is to learn something new, practice or use something known, revise knowledge and use, or to develop language, vocabulary, and representations. Many other authors have categorised the possible mathematical purposes of tasks. For instance, Doig, Groves and Fujii (2011) writing about Japanese Lesson Study categorise tasks that: directly address a concept; develop mathematical processes; delimit scope and sequence; address a common misconception. Smith and Stein (1998) categorise tasks that address: memorisation; procedures with/without connections; and 'doing mathematics'. Apart from the category 'doing mathematics', these purposes are mainly expressed as goals rather than activity. Prediger and her colleagues make distinctions between facets of mathematical knowledge as goals for mathematical learning, displayed in Table 2.

	Which part?	**Facet of knowledge**			
What? Modes of knowledge		verbalisation	distinction	meaning	conventions
	Conceptual knowledge				
	concepts	definitions	examples/counter	mental models	terms
	connections	theorems	examples/counter	explanation	conventions
	Procedural knowledge				
	procedures	instructions	conditions	mental models	specifications
	techniques	instructions	conditions		determinations
	Metacognitive knowledge				

Table 2: Task purposes categorised by Prediger in Barzel et al. (2013), simplified.

In Table 2 we see connections between the overt purpose of the task and what it means to know a concept. The contents of Table 2 are like the beginnings of the bridge built from only one side of the river. Task design needs to include theories or thought experiments about the activity that would arise from a task, and how such activity promotes mathematical learning.

From a consideration of task purpose, the following parameters can be discerned:

- the meaning of mathematical knowledge
- the aims of mathematics teaching and learning
- the local and specific learning goals for a task

Together these parameters make up the intention behind the design. Before we shift to considering the design of environment and pedagogy, we need to consider the mathematical activity that needs to be generated and shaped in implementation.

MATHEMATICAL ACTIVITY

In contrast to defining fine-grained mathematical purposes, Kieran (2004) focuses on activity necessary to learn any of the facets in Table 2 pertaining to algebra: *transforming:* mostly rule-based manipulations; *generating:* representing and interpreting situations; and *integrating:* coordinating manipulation and generation. Some describe the 'what?'; others describe the 'how?' Both depend on a 'grand' view of what learning a concept means, and/or on an intermediate theoretical view of how conceptual knowledge is learned. In Table 3 I present specific actions for mathematical enquiry, and show how they relate to conceptual aims of increasing complexity.

The connections between the focus and the suggested actions are based on the theoretical position that if learners undertake these mathematical actions, they are mentally and symbolically building complex conceptual understandings. However, *any* task design process ought to be able to connect design intentions to the likely actions of learners, and go further to explain how those actions lead to the intended learning, so this table can be seen as an example of the kinds of action, mental action, or activity that could be prompted by a task, and might then contribute to learning.

General focus	Examples of specific actions
basic actions	calculating, doing procedures, stating facts
transformative	organizing, rearranging, systematising, visualizing, representing
concept-building	comparing, classifying, generalizing, structuring, extending, restricting, defining, relating to familiar and intuitive ideas
problem-solving, proving, applying	conjecturing, assuming, symbolizing, modeling, predicting, explaining, verifying, justifying, refuting, testing special cases
interdisciplinary connections	incorporating other epistemologies, identifying variables and structures, recognizing similarities, comparing familiar/unfamiliar

Table 3: Actions for different foci of mathematical work (from Watson and Thompson, 2015)

Table 3 focuses mainly on mental activity, rather than the social, physical and dynamic activity that might appear within other frames. A sociocultural frame, in which learning is seen as participation in a system or community that is identified through patterns of discourse and participation, might focus on forms of language and interaction, the adoption of norms of behaviour in mathematical environments, and appropriation of particular tools. Thus a link between task, activity, and learning has to be made through the provision of opportunities and support for relevant patterns of action. A constructionist frame, in which learning is seen as using mathematics to complete purposeful construction tasks, might focus on trial, enquiry, and repeated cycles of engineering as the activity which connects task to learning. Any task design process has to have an underlying, coherent, connection between task, activity and learning, and therefore needs a definition of learning. Whereas there is wide international agreement about the aims of the mathematical education, I am unaware of agreement about the nature of the learning that needs to take place for those aims to be achieved. For example, in Table 3 the described mental actions could be seen as actions that give structure in a cognitive apprenticeship model of learning, in which learning is seen as becoming expert in modes of mathematical enquiry. However, they do not describe the kind of activity that contributes to the idea of learning in RME, in which bringing intuition to bear on problems is the fundamental activity. Nor do they describe the construction and refinement of utilisation schemes that is seen as learning in a dynamic, manipulable, tool environment (Leung and Frant, 2015; Sullivan, Knott and Yang, 2015). From this consideration of mathematical activity, I claim that any task design process needs to encompass:

- connections between the task as set and the mental, physical, and/or social activity that is imagined to arise while it is undertaken
- how such activity supports the expected learning.

DESIGN OF ENVIRONMENT AND PEDAGOGY

Many major task design groups provide instructions or advice about structuring learning environment and/or the associated pedagogy. This could be at a general theoretical level or at the specific level of each task. For example Brousseau, Brousseau, & Warfield (2014) pay close attention to the student's point of view (Ainley and Margolinas 2015). *Didactic contract* points to the implicit interpretations that have been shared between students and teachers about a specific type of task or knowledge. *Milieu* refers to what the student is actually dealing with: concrete objects, elementary mathematical objects. The task environment therefore includes the historical development of patterns of behaviour, as well as the provision of objects and tools. This notion, along with similar approaches using different language, problematises attempts to introduce new kinds of task without also developing new contracts, new norms, new forms of interaction, that enable learning according to new intended connections between task, activity and learning.

Task designers can indicate and illustrate particular kinds of purpose and activity. The designers of KOSIMA textbook series realised that learners needed prompts to help them organise and consolidate new knowledge in relation to their existing knowledge, and that part of their responsibility as authors might be to design suitable task-types. Thus they created and exemplified tasks that focused on systematisation, regularising and preserving new knowledge (Barzel 2013). Textbook authors try to influence teaching through provision of task-types. Advice for teachers from design teams about environment and pedagogy is often offered through teachers' guides or scripts. Swan (2007) described how to use paper, pens, materials, language forms and representations, and he tracked whether and how the didactic contract changed in classrooms of teachers who had different initial pedagogy. All participating teachers changed their practice, but the extent and direction of change, and the endstate, varied according to their starting position and other factors. Sullivan, Knott and Yang (2015), following Gimenez (2013), define five 'dilemmas' for pedagogic decision-making: the role of context; the relationship between language in the problem and in the solution; structure and openness; how content appears; and levels of interaction. These are dilemmas for designers, and also for teachers adapting tasks. For example, teachers using textbooks can increase and vary cognitive challenge through their local adaptation of textbook tasks (Watson and Thompson 2015). Pedagogical decisions inevitably influence learning opportunities and teachers rarely have time to design tasks themselves. Instead they make decisions about epistemic, cognitive, interactional, mediational, and affective suitability (Gimenez et al. 2013) while taking norms into account (ecological suitability). Smith and Stein (1998) suggest effects of teachers' different decisions when anticipating, monitoring, selecting, sequencing, and connecting the learning effects of tasks. Kullberg (2013) shows how, even where there is agreement of intermediate theories and shared planning, language and gesture by the teacher can lead to varied learning, whether deliberate or implicit.

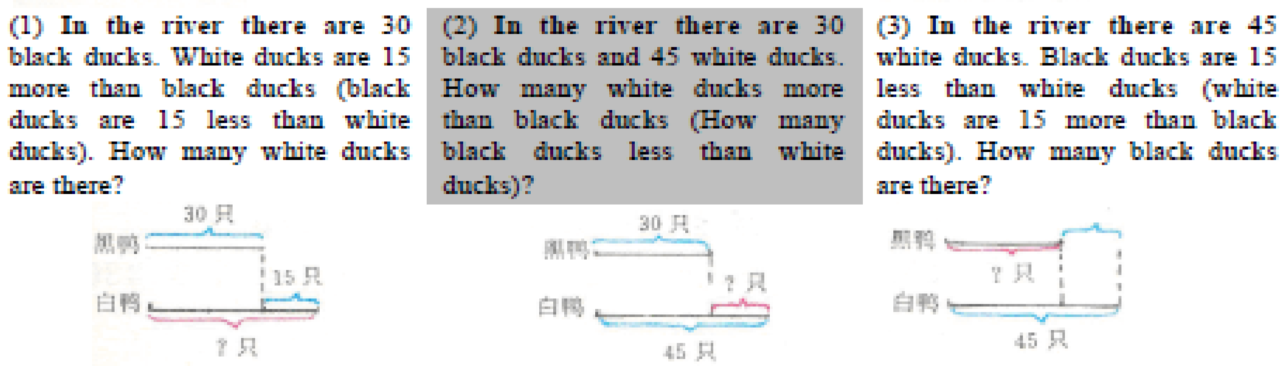

Figure 1: Three of nine situations to be compared (from Bartolini Bussi, Sun & Ramploud, 2013)

Overarching cultural expectations act as intermediate frames that influence implementation, as is illustrated in the ICMI Study when Italian and Chinese teachers presented the same comparison task to their students involving nine similar situations (Figure 1 shows three of these) (Bartolini Bussi, Sun & Ramploud, 2013). In China nine cases were presented at the same time, and students classified them according to similarities and differences. Italian teachers split the task into several parts, asking students to invent similar problems, orchestrated discussion. For them, discussion was essential for learning as participation in a semiotic culture, whereas for the Chinese learning was discernment of concepts through becoming familiar with variations in their representation and transformations.

Further parameters arise from these considerations:

- expectations about learners' point of view and likely responses
- assumptions about available tools and materials to support activity
- expectations of interaction, language and communication
- local norms of teaching
- the capacity to inform teacher decisions about typical dilemmas and suitability

A further idea that emerged in considerations of tool-use during the Study is *discrepancy potential* (Leung and Frant 2015). This labels the way tool use might model a mathematical idea in ways that do not match the concept closely, so that some discussion is necessary to understand outcomes. This idea, I claim, can be extended to other representations in language, symbols and diagrams. Space that emerges between representational outcomes and mathematical meaning can intentionally or accidentally introduce disturbances that, with discussion, can demonstrate the need for the underlying mathematical concept and its behaviour. This leads me to include as additional design parameters:

- the potential for uncertainty and need for contingent action.

CONCLUSIONS

For some well-established and research-informed design teams, the parameters I present may seem obvious, but my aim here has been to bring them together in one place to structure and inform future research - not that everyone needs to work elaborately on every feature, but that a task design study could (and maybe should) be able to present a commitment to each parameter and thus build on research about design, intention, implementation, and learning. This is not to suggest that teachers designing and adapting their own tasks need to be articulate about all the parameters, but that design teams should be so, and researchers should be so, teasing out the tacit knowledge of teachers and other practitioners.

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