A critical look at some of the references that were used for the Early Career Framework Document from a perspective of mathematics teaching.

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Great care is necessary when attempting to apply research from cognitive psychology to teaching. Technical terms need to be well-defined. The context and scope of the studies on which the findings rely need to be relevant to continuous classroom learning and the relevant subject matter. The detail of studies can reveal limitations in scope and meaning that can be lost when trying to apply conclusions to practice. The following comments expand on these statements.

Abbreviations:

ECF Early Career Framework (https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/773705/Early-Career_Framework.pdf)

PS problem-solving

WM working memory

CL cognitive load

1. Deans for Impact: The Science of Learning

This document was used to support the ECF sections 2 & 3 and includes a section on **Cognitive Principles.**

‘To learn, students must transfer information from working memory (where it is consciously processed) to long-term memory (where it can be stored and retrieved). Students have limited working memory capacities that can be overwhelmed by tasks that are cognitively too demanding. Understanding new ideas can be impeded if students are confronted with too much information at once’.

1.1 Working memory is used differently by different authors (see the paper referenced by N.Cowan in the ECF). Some conflate short-term memory with working memory. Short-term memory is very short – a few seconds. In this document it is used more usefully to include conscious processing, but that use means that much of the research about capacity limitations may not apply.

1.2 Throughout the literature from cognitive science, ‘learning’ is taken to mean a change in long-term memory. In most research, this is taken to mean facts and their associated processes. In some this is more explicitly given as structures and their associated processes, i.e. schema. In mathematics this is useful, but not the whole of what can be described as learning. We also learn the modes of enquiry specific to mathematics, and the forms of notation and recording that enable us to 'chunk' complex tasks so that they do not make excessive demands on working memory. I suspect this is true for all subjects. It could be argued that, although modes of enquiry are behaviours, they are also lodged in the long-term memory. However, subject situated behaviour is not what is researched – factual and procedural memory are what is researched.

1.3 Evidence for the final sentence is referenced to Sweller 1988, see next section.

2**. Sweller, 1988.**

2.1 Sweller's research is mainly about mathematics, so it is unclear how far it can be generalised across all subjects. It is also unclear how far it can be generalised across mathematics. He is interested in schema acquisition (see 1.2 above). For mathematics, recognition of familiar structures and memory of their associated processes is important, but it is not the whole of mathematics learning. We also have to work with unfamiliar structures. He uses the 'conscious process' meaning of working memory, which is useful.

2.2 Sweller argues that cognitive load during problem solving affects learning. Cognitive load is the number of processes that have to be done to solve a problem. 'Problems' in Sweller's corpus of work are what others might call questions or tasks. e.g. find the length of a side; calculate the moment of a force; find the roots of an equation and so on. Learners are expected to become familiar with the type of task, its configurations and positions, and sequences of moves that would solve it, i.e. the schema. Similar research has been done for chess moves, electronic circuitry, baseball configurations, and so on. In mathematics however, 'problem' has two other meanings: (a) an ill-defined realistic situation that might yield a range of solutions once it has been mathematised and (b) a puzzling question that arises as a result of a mathematical phenomenon, e.g. how does the number of real roots of a polynomial relate to the coefficients? Or 'Can we subtract a larger number from a smaller number?' Results from Sweller's research cannot be used as arguments for and against all the kinds of problem-solving that constitute work with mathematics.

2.3 He posits two types of problem-solving: one is goal-directed - there is an answer to be found; the other has no specific goals. In UK terms these latter type would be called exploring or investigating; the example he gives is: 'calculate the value of as many variables as you can in this situation'. The first kind generates a higher cognitive load in multi-stage problems (and seems to be also more error prone) because at each stage of solving the end goal has to be taken into account: 'where am I and where am I trying to get to?'. The second kind has a smaller load because there is no goal, so no means-end analysis to be undertaken. This is contrary to some of the ways in which CL theory is being used to justify the use of closed tasks rather than explorations, although if there is only one unknown both kinds generate similar loads.

The model he uses to compare these two kinds of PS is based on the assumption that equations/statements are being set up to describe a situation and processes involve a chain of equations, either algebraic or geometric or logical etc.

2.4 The argument that cognitive load is higher for goal-directed PS seems to ignore the fact that, because stating a situation, intermediate steps and multiple manipulations might be involved mathematics has developed symbolism and diagrammatic representations to enable chains of reasoning to be recorded and hence reduce cognitive load, because not all processes have to be carried mentally. These methods of reducing cognitive load by chunking and representing have taken place over hundreds of years and constitute much of the school mathematics curriculum. Sweller does mention this in passing (pencil and paper is an 'external memory source') but it does not enter his discussion of working memory – nor anyone else's as far as I can find. The whole literature on cognitive load, even though much is about maths (or arithmetic in many cases), ignores the invention and use of symbolism to enable keeping track of reasoning. So in a sense this whole argument about WM is spurious when applied to mathematics, but in another sense his argument supports the use of open-ended questions.

2.5 He then goes on to ask about how schema acquisition can take place alongside PS. Some of the research he mentions uses 'dual task' research. He regards PS and schema acquisition as separate tasks. Dual task research asks whether someone who is busy problem solving can also do another task alongside. In some of this research the second task has nothing to do with the first and could be understood as a distraction or interruption (fortunately in maths we can record what we are doing so may not be so distracted). In other studies the second task is connected in some way, but not part of the PS process, e.g. remembering task steps in a different order. In Sweller's paper, the study asks students to remember the givens and solutions of the problems as well as solving them. Unsurprisingly, they concluded that PS interfered with the memory task. Of course. This ignores the classroom realities of the use of recording methods, of post-task discussion reflecting on methods, and the other ways in which familiarity with structure and associated processes develop through several experiences, language use, and so on. Research on isolated tasks, rather than sequences and multiple experiences that develop familiarity and recognition, cannot be used to justify particular reductions in cognitive load, nor a need for step by step methods of schema acquisition that ignore recognition, generalisation, intuition and insight.

2.6 This paper, while having a lot to offer cognitive psychology, has little authentic to offer about the *teaching* of mathematics, and even less for other subjects that are less structural, and the section of 'The Science of Learning' that depends on it is an unsafe basis for part of the ECF. It ignores the role of recording in mathematics, and the roles of reflection, discussion, language and multiple experiences in learning that support what Sweller calls schema acquisition.

3. The ECF offers a paper by **Adesope et al.** (2017) that shows that **practice tests before a formal test help success in the formal test.** This would not be a surprise to teachers since learning how to do particular tests will be in their own experience and is common practice in schools. However, this paper does not offer anything that supports the regular or frequent use of practice tests. It is not a useful source for 'making progress' unless all this means is 'doing well in tests'.

4. The ECF uses a paper by **Agarwal et al.** (2017) to support suggestions about promoting good progress. '**Benefits from retrieval practice** are greater for students with lower working memory capacity'. The title does not say what this is greater than, but it turns out to be 'greater than for students with higher WM capacity'. The test was carried out by asking students to remember 110 general knowledge facts for a test. We are not told if the items had any connection to their continuous study, nor if there were any connections between the items. We are told they are 'trivia' factual questions. Although the results might be interesting for knowing more about WM they cannot support an argument for retrieval practice in the context of promoting progress in continuous study in which facts have some meaning and their accumulation has some conceptual developmental purpose, which the tools used in this study do not have.

5. **Dunlosky et al.** (2013) is offered to justify the promotion of good progress through teaching that supports particular learning techniques. However, the authors say the paper should be used to 'encourage students to use **appropriate learning techniques'**. Its use to support certain teaching recommendations needs to be circumspect.

    5.1 The techniques investigated include elaborative interrogation, self-explanation, summarization, highlighting (or underlining), the keyword mnemonic, imagery use for text learning, rereading, practice testing, distributed practice, and interleaved practice. Of these, some are found to be of 'low utility' and some of 'high utility'. 'Low utility' does NOT mean ineffective; it means that the technique is not widely useful for all kinds of subject material and has limited applicability. 'High utility' means that there is evidence that it is useful across multiple kinds of subject learning. This study seems to be used in the ECF to support the spacing of practice. I will look at this in more detail below but firstly would say that it gives insights into a wide range of learning techniques in educational contexts and therefore a wide range of possible teaching aims and purposes.

    5.2 Their review of literature on '**distributed practice'** refers mainly to factual knowledge and trivia and shows that the duration of retention of knowledge relates to some extent to the length of time between study sessions – the longer the space, the longer the duration of retention, given a learner's willingness to learn – up to a year's space for ideas that need to be retained for 5 years. This reminds me of Bruner's suggestion of constructing a spiral curriculum so that learners return to the same domain of knowledge several times during their school career. Because of the cumulative nature of mathematical concepts, this is inherent, but perhaps implicit, in most mathematics curricula. This inherence can be damaged by fragmented curriculum statements and testing regimes. The main study they offer from an educational context involves vocabulary acquisition which is achieved through multiple kinds of experience involving reading, writing, defining and using in sentences. Teaching was spaced a week apart and this was found to be more effective than spacing a few minutes apart.

5.3 Their review of the literature on **interleaving** suggests that the positive effects of this are well known in motor learning, but only recent in cognitive tasks, for which evidence in psychology is still thin. Nevertheless, they say classroom studies show 'some promise' and give a mathematical example. Two groups of students were given four slightly different but related mathematical examples and had to work through for themselves exercises that used the four slightly different approaches they had seen in the teaching. One group were given these all at once, the other group were given the four variations separately in four sequences of teaching and doing. The group that were shown all four before doing a mixed exercise did better in a later posttest than those who had been given the four variations and their exercises in four consecutive episodes. In mathematics this **contradicts some prevalent advice to teach in small steps** and supports the use of variation to provide examples that, through contrasting sameness and difference, lead learners to engage with underlying mathematical structures and their associated processes.

5.4 While this paper has some usefulness for teacher knowledge, its use of to support 'promoting progress' confuses research on 'appropriate learning techniques' with recommendations for teaching. The research on spacing and interleaving in mathematics is thin but interleaving would be a way to describe some well-tried uses of variation theory and *bianshi*.

6. **Sweller (2016)** This paper is used to connect cognitive psychology research to recommendations for 'instructional design'.

6.1 As above in section 2, some of the studies to which he refers are of isolated bits of knowledge not in a context of continuous educational experience; problem-solving is taken to mean finding unknowns; cognitive load is measured by the number of processes required; learning is taken to be schema acquisition; and the role of recording methods and language to chunk knowledge into familiar structures and their associated processes is underplayed. For example, he says: 'when faced with an algebra problem such as (a+b)/c = d, solve for a, the best first move is to multiply out the denominator… a domain-specific skill that applies only to a limited class of algebra problems' and he uses this as an example of something that has to be taught. This assumption ignores that such a problem can be seen as situated in a class of multiplicative relationships for which the juxtaposition of terms could be familiar and 'chunked' and the 'best move' deduced by reasoning rather than it needing to be taught explicitly. This does not contradict his attention to working memory issues, but does flag up a warning that **what counts as evidence for the purposes of cognitive psychology does not automatically provide advice for teaching mathematics.** Indeed, his own work on schema acquisition would support the teaching of generic multiplicative structure rather than individual algebraic moves.

6.2 The paper summarises several results from his corpus and the related work of others. The **split-attention** effect says that having to split one's attention between two different sources results in a high cognitive load. **Studying worked examples** where the answer is known is more effective than working through examples because the method can be focused on instead of 'getting the answer'. Having to cope with **unnecessary items as well as necessary** ones increases cognitive load. All these observations and research results are no doubt true but doing mathematics well includes: being able to identify necessary and unnecessary information; coordinating several sources of information (such as graphs, diagrams and symbols); tackling unfamiliar problems using prior knowledge and exploratory skills. To focus on reducing cognitive load can result in reducing mathematical experience to factual and procedural knowledge.

6.3 Sweller (2016) is useful to remind teachers to anticipate inherent difficulties that are due to cognitive load, but slavish use to reduce cognitive load might lead to a limited and simplistic mathematical experience for learners. By contrast, the use of mathematical tools that 'chunk' knowledge through the use of symbols, diagrams, language and the development of familiarity are inherent in the mathematical canon and the ways in which its concepts develop.

7. **Wittwer et al.** (2010) The title of this paper says that it is about 'example-based learning' which might sound attractive to mathematicians, since mathematics is met through examples from which generalisations and relationships can be understood. However, it refers only to research about the use of worked examples in teaching and hence only to learning procedures. As such it is of limited use for mathematics teaching and it is hard to see how is might be useful for other subjects.

8. **Rosenshine** (2012) This paper gives plenty of advice for teachers, claiming that it is supported by research in education and cognitive psychology. Curiously, some of it

contradicts what is offered by other cognitive psychologists. For example it does not seem to support interleaving and instead supports a process of teaching in micro-steps. It is influenced by Rosenshine's own well-known preference for a particular kind of teaching, which can be found at: http://www.centerii.org/search/resources/fivedirectinstruct.pdf. Personally, I can see roles for everything he suggests in certain circumstances, but not as the sole methods of teaching.

In this paper I have not entered the debates about direct instruction and discovery learning – both have multiple meanings that can overlap, both might be done well, both might be done badly, different aspects of mathematics learning require different teaching approaches.

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