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RED HERRINGS: POST-14 'BEST' MATHEMATICS TEACHING AND CURRICULA

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ABSTRACT: The Smith Report has generated central questions about the mathematics education of UK adolescents. This paper highlights the close match between the goals of school mathematics, adolescence and exploratory pedagogy. This is contrasted with the prescriptive nature of current regimes. In particular, without careful attention to pedagogy it is possible that the introduction of different pathways may lead to a failure to achieve the outcomes desired by employers and universities, and to inequity in provision for students.

Keywords: low attainment in mathematics, mathematics teaching methods, secondary mathematics teaching, post-14 mathematics curriculum

1. Introduction

In this paper familiar questions are briefly raised about the effectiveness of current regimes for those who presently fail to achieve government targets at Key Stage (KS) 3 (25 per cent) and at GCSE (50 per cent). Submissions to the recent Post-14 Mathematics Inquiry (Smith, 2004) are used to collate the views of employers and universities about appropriate mathematical preparation for adulthood. Research in mathematics education shows a strong match between outcomes desired by employers and exploratory, collaborative, teaching methods. Some features of adolescence are then described, and a strong match between these and the messages from employers, universities and pedagogy research is drawn. Finally, the argument is made that current plans to provide differentiated pathways from 14 is likely, within current regimes, to reinforce inequity and alienation which could be avoided with proper attention to pedagogy.

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Why 'Best Methods' are an Issue

In the UK, attention has recently been directed towards the role of mathematics in the school experience of students aged 14 and over. As part of a wide-ranging review of related issues, Smith (2004) posed questions about effective pedagogy. In particular, there was a desire to identify, if possible, 'best' ways to teach mathematics. The existence of a strong frame for the mathematics teaching of students up to 14 makes sense of this desire. The well-resourced National Framework for Mathematics for Years 1 to 9 advises on schemes of work and teaching methods, providing a large amount of training and supportive material such as lesson plans, worksheets, task ideas and so on. Teachers and schools often interpret the Framework as compulsory and, indeed, those who deviate from it may be given a difficult time by inspectors, especially if the school already has problems with underachievement. Within a short space of time the ways of teaching recommended by the Strategy have become 'the way we teach mathematics' throughout England. The evaluation team from OISE has commented positively on the effectiveness of the Strategy in terms of influencing the organisation of mathematics teaching (Earl *et al.*, 2003). It might be thought, therefore, that there *is* a best way to teach, and that the Framework embodies it for Years 1 to 9. Thus finding a similar strategy for older students seems to be a meaningful goal.

However, while crediting these strategies with a general improvement in teaching and learning, recent Ofsted reports indicate that uncritical adoption of some of the materials has led to teachers offering too limited a time to learn, moving through the curriculum too quickly and with too frequent changes of topic for lasting learning to take place, especially for the lower attaining students. The programmes of study offered to students can be too limited, too early, thus restricting opportunities to learn (Ofsted, 2004a, p. 12). Some teachers have stuck too rigidly to a given lesson structure without imbuing the tasks they offer with purpose (Ofsted, 2004b, p. 22; 2004a, p. 7).

The effectiveness of both the National Numeracy Strategy and the associated drive towards raising standards, as measured through the national test results, shows an annual rise in proportions of children achieving certain desirable levels. These rises are currently levelling off and a significant number (about 25 per cent at KS3 and about 50 per cent at GCSE in mathematics) still fail to achieve the minimum acceptable levels (Ofsted, 2004a). It would be possible to say that all that these rises show is improved effectiveness in getting students to pass the national tests, and we do not know how far the rises represent

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test-training and how far a real improvement in mathematics. Indeed, the problem of defining 'effectiveness' gets passed on to the word 'improvement', for if we see improvement as measured by achievement in a certain test then we can talk about it meaningfully, but if we want improvement to mean something more than test results then we have to look at the match between the test and the educational goals. A longitudinal study funded by Leverhulme has found that Year 4 students have only undergone a three per cent rise in scores on a standard test, administered annually, and the biggest rise took place in the year before the Framework was launched (Brown, *et al.*, 2003)!

The setting of acceptable levels of knowledge of mathematics for different ages is always arbitrary and has to relate to other decisions about development. For example, one could say that students need to be able to handle money by a certain age for social reasons, or measurement for cross-curricula reasons, or abstract reasoning for intellectual reasons, or geometrical pattern making for aesthetic reasons. There are also some hierarchical features of mathematics such as knowing about place value before multidigit multiplication, or ratio before trigonometry, but these are more contentious than might be expected – it is not necessarily the case that learners learn mathematics in an order which makes sense to curriculum planners or logicians (Denvir and Brown, 1986; Kieren *et al.*, 1999).

When we look at secondary education, development of adolescents may not be seen as merely academic; appropriate goals might relate to future employment and citizenship, hence to discuss effective pedagogy there has to be a clear statement of *all* the goals of teaching. And even if development *is* seen as wholly academic, there has to be agreement about the nature and appropriateness of the subject under consideration. All research about pedagogy in mathematics is influenced by a 'nature and purpose' standpoint, since these give meaning to 'effectiveness'. A pedagogical approach can be said to be effective if it achieves its purpose through promoting learning and development of a specified nature.

2. What School Mathematics is for

There are variations in public views of the possible nature and purpose of school mathematics teaching (Ernest, 1991; Niss, 1996). For the purposes of this paper I shall summarise these into three main views:

- School mathematics can be seen as a collection of useful skills and procedures which have to be learnt. This calls for a training pedagogy, focusing on memory and accuracy. Nearly all mathematical

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procedures at school level, even secondary school level, can now be done by machines, but there is a folk investment in having such skills. There is a need to understand the limitations of such skills, and when memory and fluency are essential.

- School mathematics can be seen as supporting the early stages of becoming a numerate professional mathematician (scientist, engineer etc.), providing a basis for higher mathematics. An appropriate pedagogy would have to take into account what needs to be fluent for later use in more complex situations (such as multiplication facts, transformation of equations and expressions, some trigonometric identities, some definitions and so on), and what needs to be understood structurally for later conceptual development, flexibility and application in more complex contexts (also such as multiplication facts, transformation of equations and expressions, some trigonometric identities, some definitions and so on). Thus the 'same' content can be framed in both these ways and cannot neatly be split into types for different purposes.
- School mathematics can also be seen as a tool for application and problem-solving in order to participate fully and in an informed manner in society. Thus what is learnt has to be available to be used flexibly in unfamiliar situations and has to be understood enough to be of use intellectually and practically. An appropriate pedagogy would involve seeing situations as arena for quantitative, structural, graphical and spatial exploration, selecting mathematical tools and evaluating the outcomes of their use. This was the view taken, by and large, in the Cockcroft Report (1982), to which a wide constituency of industrialists, politicians, mathematicians and educationists contributed. For example, in paragraph 12 of the Cockcroft Report the purpose of mathematics is seen as 'preparation for adult life, for employment and further study, appreciation and enjoyment of mathematics and the role it plays in development of civilisation'.

These views of school mathematics are mutually compatible if one sees any new mathematical situation as an arena for exploration. Structural understanding, flexibility and evaluation are required for all these views, and fluency in some areas in order to free intellectual capacity for dealing with complexity is also common.

Submissions to the Post-14 Mathematics Inquiry showed that informed identification of desirable outcomes of school mathematics has barely changed since the Cockcroft Report. These varied somewhat according to whether the submissions were from individual mathematicians, employers or universities, but there was considerable

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agreement. For example, there was widespread accord between universities and major employers that depth of understanding, non-routine problem-solving skills, and fluency of that which can usefully be fluent are appropriate outcomes of school mathematics. Willingness to deal with complexity, to engage with multi-stage tasks and new situations, either within pure mathematics or within the workplace, was a commonly stated need. There were pleas for basic skills to be understood and fluent, but what was identified as 'basic' varied. For universities, 'basic' skills were algebra, calculus and trigonometry; for employers, 'basic' had a range of meanings from 'arithmetic without calculators' to 'problem-solving skills with appropriate use of ICT' and a broad mathematical literacy which encompasses: interpretation and use of different representations of data; data-entry and monitoring; related communication skills; recognition of errors and anomalies; knowledge of what, how and when to calculate; use of relevant degrees of accuracy and plausibility. Keen, critical analysis of mathematics, and of the use of mathematics, such as awareness of error and of the implications of results, is seen by universities and employers as more important than grades, because current assessment regimes are not seen to be telling the story required by the users of the output of the education system (Smith, 2004).

Employers perceive a lack of both mathematical skills of the kind described above, and of hybrid skills such as mathematics with communication and/or teamwork. The needs of employers are supported by systematic research into the mathematical needs of several major employers in various sectors which has led to the identification of important components of 'mathematical literacy' in the workplace (Noss *et al.*, 2001). While such literacy is always anchored in the work situation, motivated by having real problems to solve with real data, some generic skills are involved, such as: ability to perform analytical, flexible, fast and multi-step calculation, estimation in context, modelling of variables and relationships.

3. Mathematics Pedagogy and Attainment

Pedagogy concerns the actions teachers take, the intentions behind the actions, and how these relate to the progress of learners within a mathematical environment. Good pedagogy is characterised by an enhanced, integrated relationship between teachers' intentions and actions on the one hand and students' learning and development on the other.

In discussing methods for teaching mathematics it matters which view of the subject is taken, because a choice has to be made between

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following remembered instructions, and learning with a level of understanding which allows later use beyond what is available from memorised procedures. Skills, knowledge and confidence in approaching and solving non-routine mathematical problems, within and outside mathematics, can only develop through extensive experience over time, so that appropriate habits of mind become the norm. For much workplace problem solving, learnt heuristics suitable for classroom are unhelpful, and the more complex approaches associated with mathematical modelling, such as identifying and controlling variables through setting up and manipulating algebraic and graphical representations, can take several years to develop through experience. Understanding, flexibility and fluency with mathematical concepts and manipulations also need to develop over time.

Wearne and Hiebert (1988), in a long-term project to teach mathematical understanding, found that primary school children who had been taught procedures found it much harder to *then* understand the meaning of what they had to do than those who first learnt conceptually, so that procedural knowledge emerged within conceptual learning. However, Rittle-Johnson and Siegler (1998), in reviewing a wide range of relevant literature, found that the relationship between procedural and conceptual understanding varied between mathematical subtopics, sometimes the two developing adequately together and sometimes separately. They found significant differences even within procedural arithmetic; that is to say that there was not even one 'best' order for teaching all computational skills.

This degree of variation within one subject makes good sense when consideration is given to the different cognitive demands made by, for example, recalling the procedures of arithmetic, generalising number structure, visualising three-dimensional shapes, interpreting statistical diagrams, and imagining very small objects in calculus. Broadly, learning mathematics requires, at every level, the development of knowledge of concepts, techniques, notations and relationships; recognising them in familiar and unfamiliar forms; recalling facts, names, procedures; using procedures fluently and accurately; the ability to shift between methods and representations; applying knowledge to solve problems, possibly transforming it to do so; and creating generalisations, abstractions, images and methods. In order to achieve this complex knowledge, learners must have opportunities to do appropriate things in mathematics lessons: sort, classify, structure, abstract, generalise, specialise, represent and interpret symbolically and graphically, justify and prove, encode and decode, formulate, communicate, compare, relate, recognise familiar structures, apply and evaluate applications, automatise.

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There is, therefore, remarkable professional agreement among mathematics educators, universities and employers about the need for complex mathematics teaching which achieves these multiple purposes in an integrated way. Ironically, governmental efforts to improve attainment in mathematics, which had initial intentions that largely matched those of employers and universities, have effectively prescribed methods of teaching a very full curriculum in ways which work *against* such long-term education into ways of acting mathematically in and with the world for many students. In particular, low-attaining students fall further and further behind as they are given 'catch-up' and 'booster' programmes which focus largely on technical skills, which could be done with ICT, instead of being helped to think in the more sophisticated ways which would better prepare them for adulthood. The same demands for technical repetition are not made on higher-attaining students, whose intermittent inaccuracies become less and less important as the concepts with which they work become more complex. It may have been better to heed the more humanistic approaches of education based on 'morally or educationally defensible principles' (Davis, 1999, p. 400) than the technical rationality which has been embraced instead. Even though there has been agreement about ends there has not been shared understanding about the educational means by which these ends would be achieved.

In studies of the type reported by Rittle-Johnson and Siegler, and in any studies purporting to show cognitive development of students who have already had a long time in school, the pedagogically interested reader is always left wondering 'what was the teaching like?'. We cannot assume that teaching is uniformly chalk-talk-and-practice if we are told nothing about it. Typical secondary school lessons, as Hiebert and others have shown (2003), vary hugely from country to country and there is nothing obvious about typical teaching styles in countries whose students are successful in TIMSS and PISA-OECD tests, if these are used as yardsticks for success. Most startling is the contrast between Japan and Hong Kong, both high achieving countries in international tests, in which typical lessons vary from whole-class interactive solution of complex problems (Japan) to silent individual practice of decontextualised procedures (Hong Kong). Both, however, have a collective approach to what constitutes achievement; the belief is that all learners can succeed. Aspects such as teacher clarity or proportions of time spent on different parts of lessons do not seem to relate to overall achievement, possibly because there are so many other factors to be taken into account. What makes the difference to attainment in the studies of Margaret Brown and her colleagues is

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not the format of the teaching but the ‘quality’, seen as including the nature of learner responsiveness, appropriateness and challenge of examples used, explanations, interactions, discussions, tasks and exercises (Brown *et al.*, 2001) But it has been known for a long time that the teaching style is not likely to be the major factor in the nature of mathematical achievement, because ‘approaches to teaching of a particular piece of mathematics need to be related to the topic itself and to the abilities and experience of both teachers and pupils. … Methods which may be extremely successful with one teacher and one group of pupils will not necessarily be suitable for use by another teacher or with a different group of pupils’ (Cockcroft, p. 242), a point elaborated and illustrated more recently by Davis (1999).

Evidence of the variety of practices which result from teachers adapting to fit imposed styles is given by Tanner and Jones (2000). They conducted a long-term study of teachers who were adapting to using investigative methods of teaching and were as fully involved in the research as teachers can be. The teachers enacted very different roles: in one case, students were rigidly channelled to go through the thought processes developed by the expert teacher, in another case, students were seen as the experts, able to make their own choices with appropriate support and discussion with the teacher. Those in the latter group developed stronger conceptual learning, and were better able to transfer their knowledge into very different situations than those in the more teacher-centred classrooms. The professional development context did not lead to teachers adopting similar practices.

In the TIMSS seven-nation video study, high scores in international tests are associated with pedagogies of pursuing tasks and arguments which make explicit connections within mathematics (Hiebert *et al.*, 2003, p. 104). What high-achieving countries appear to have in common is the conceptual complexity of problems posed to students, the avoidance of simplifying such problems once posed, and the intensity of engagement of students. Whether engagement is through effortful bookwork, collaborative groupwork or whole class interaction does not seem to matter; it is the extent to which the locus of knowledge generation is with the learners which makes the difference. When professional mathematicians viewed videos of typical classes from several high-achieving countries they saw, as well as the aspects described above: explicit use of deductive reasoning; explicit development of mathematical rationale; the presence of generalisation in many lessons; and coherent progression of mathematical ideas within lessons. The sample for this part of the study

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could not yield statistically significant generalisations, but the findings did indicate that the overall important features were *not* class size, organisation, lesson structure or superficial features of the tasks. A further similarity they found was in the nature of the lesson content – what was taught to nearly all students in high-achieving countries was simply more mathematically advanced than elsewhere.

What can be learnt from these studies is that teaching which involves all learners in activities requiring complexity, reasoning and knowledge generation seems to get higher results in international tests, and furthermore the core similarities of the teaching match the wishes of UK employers and universities for the outcomes of school mathematics. Yet lower-attaining students are commonly offered limited, purposeless tasks (Boaler *et al.*, 2000; Hart, 1998).

4. The Engagement of Adolescents with Mathematics

The particular context of 14–19 learning offers special challenges. By this age, some students have managed to be successful with procedural knowledge only, and are beginning to face situations in which what they thought they knew turns out not to be so helpful; others may have only managed to generate their own *ad hoc* approaches to arithmetic which do not provide the structural understanding of number required to inform algebraic learning; elementary ways of transforming shapes and their properties may have to be re-thought; fractions can no longer be seen only as slices of pizza because this image is unhelpful when dealing with ratio and probability; naïve additive approaches to numerical understanding have to be supplanted with multiplicative ones. Ways of coping which may have helped students in the past have to be re-assessed and restructured, and all this happens at puberty, at a time when social and emotional lives, and senses of self, might also be unpicked and restructured. In addition, the after-effects of teaching-to-the-test for KS3 SATs and the beginnings of a drive towards GCSE may combine to ensure that some students get very lost and frustrated with the subject, particularly when 50 per cent of them are unlikely to get grade C or higher.

Changes in the nature of school knowledge which occur at the same time as changes in maturity are not only an issue in mathematics. In general students need to experience personal change and meaning-making to 'complete' their learning, and students who do not do this without prompting, coercion and support are disadvantaged because they are then vulnerable to changing circumstances out of school (Watkins and Mortimore, 1999). It is helpful to see this vulnerability as acting out in classrooms as well. The combination of

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adolescent conformity and rebellion, the need to work in a more abstract way in mathematics and the insecure foundation provided by superficial understanding of some key areas of mathematics, plus the lack of any personal power in knowing whether your maths is right or wrong, is a dangerous soup. Hence the problems of engaging students with mathematics and developing their conceptual understanding are the same problem: how to help adolescents develop as mathematical thinkers, and to develop their sense of self as mathematician within other aspects of their identity.

Young and Lucas (1999) describe typical shifts in the pedagogy of most subjects as learners grow older. These include shifts from teacher-centred to learner-centred organisation; teacher-authority to learner-enquiry; insular to connected knowledge. These shifts parallel what is being said about the learning of mathematics, in that the inherent authority of mathematics, the need for problem-solving approaches, and the need for discussion match a shift to more autonomous organisation. Sadly, in mathematics some of the opposite shifts tend to be the case, in that the demands of abstract work lead to an increased focus on the teacher as authority, so that the older you get, the more likely it is that your mathematics teacher will be mainly telling you things, especially if you are in a low attaining group (Hart, 1998; Ireson and Hallam, 2001, p. 37).

During adolescence, the mathematics curriculum becomes more formal, rigorous and complex and new ideas come thick and fast. Mathematics for this age group is characterised by shifts towards abstraction and formalisation, away from situations which yield to concrete, numerical, visual and ad hoc reasoning (Tall, 1991); this may seem to make it less relevant for obvious uses outside school. The benefits of being able to generalise structures and hence become better thinkers and problem-solvers are not immediately obvious, and adolescents are typically not good at seeing very far ahead. The traditional response of the UK education system has been to alter the curriculum. Thus, historically, formal geometry and algebraic manipulation were excluded from the curriculum in the 1980s and have only recently been re-included. Debates around these changes illustrate embedded positions of the debaters. On one side, proof and complicated algebra can be seen as élite language privileging forms which express external hierarchical power of university mathematicians and are divorced from the real concerns of most learners (Confrey, 1999). On the other, university mathematicians point to the disadvantage experienced by students who have not had these experiences throughout their formative years (Smith, 2004). For a variety of reasons, rigorous exploration of abstract ideas was reduced

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to an investigational algorithm in order to make it teachable during the 1990s. Currently, the interpretative reasoning required with statistical exploration is proving hard to teach so this, too, may have to leave the core curriculum (Smith, 2004, p. 85). Yet at each of these historical moments there have been teachers who *have* taught most adolescents successfully to achieve these 'impossibilities'. For them, the observation that students *don't learn* triggers pedagogic action, rather than inaction, despair or attempts to control learner behaviour even more closely. For them, the argument is not about privilege or the needs of learners determined by their existing social status – it is about the rights of all students to have access to mathematical culture in a broad sense, much as one might hope that adolescents might at least know about the existence of opera, Picasso, religion, geometric proof and socialism, even if they reject these options at some stage (see for example Ollerton, 1993; Ollerton and Watson, 2001).

One outstanding feature of UK mathematics teaching which endures through all changes in school organisation, assessment systems, policy and imposed 'normal' practice is the segregation of students from each other in order to be treated to a different curriculum, with different expectations, and that these differences are discussed in terms of fixed deficiencies for the most vulnerable and potentially troublesome adolescents. Thus, whatever the intended structures of the system, the structures activated at classroom level disenfranchise certain students by limiting the curriculum offer. The system is so wedded to this idea that the recent interest in descriptions of multiple intelligences is used in some schools not to challenge existing notions of fixed abilities (Hart, 1998) but to confirm teachers' grouping decisions by showing that those already consigned to lower sets are mainly 'kinaesthetic' learners and hence 'need' kinaesthetic engagement, rather than sensitive induction into abstract engagement that their peers in other groups might be getting. As Broadfoot points out, the way the education system sorts people out is so insidious that by and large students and parents are lulled into acquiescing to different treatments which are justified by deficiency arguments and spurious reference to the competitiveness of test results rather than proficiency (Broadfoot, 1996, p. 5; Hart, 1998; Watson, 2001).

Current debates about the provision of diverse pathways in mathematics from 14 to 19 make examination of curriculum differentiation urgent. Access to intrinsically difficult mathematical ideas will be granted differently by those who are driven by ideologies of choice, or of entitlement, or of needs.

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Choice: Adolescent as Consumer

In intellectual life adolescents feel they lack choice and authority, yet in economic life they are beginning to, and indeed are encouraged to exercise a great deal of choice and authority. Mathematics quintessentially offers intellectual choice and absence of adult authority, because the ultimate authority for mathematics is within itself, and not with the teacher or the textbook (Hiebert *et al.*, 1997, p. 40). Offering adolescents opportunities to make choices and to check them out within mathematics, rather than depending on the authority of others, seems to make a difference to both engagement and understanding. This is, in my view, what is usefully meant by 'relevant' – relevant to their current emotional and developmental states – proximally relevant. Scaffolding can be provided by presenting problems which are 'sufficiently close to their current knowledge to be assimilated and ... sufficiently different in order to force them to transform their methods of thinking and working' (Grugnetti and Jaquet, 1996, p. 616).

The recognition of society and their own place within it which develops strongly in this age group is relevant to this discussion, because the wideness of the world and the possibilities in it, and one's inclusion in or exclusion from it, greatly affect one's choices. In addition, adolescent interest in the behaviour of social groups encourages conjecture and generalisation, at the same time as one is expected to conjecture and generalise in mathematics. Choice in life can be paralleled and modelled by choice in any classroom, and the dependence of mathematics on its own internal structures for validity makes it an ideal 'safe' arena for learning about dealing with the outcomes of choice.

While choice can be empowering, or a way of promoting engagement, choice at 14 of an appropriate mathematics pathway is not likely to be a fully informed choice. By that age, at least 25 per cent of students have had a very limited and negative experience of mathematics. At best they are likely to be attracted to vocational pathways because of supposed employment prospects, at worst they are likely to be assigned to them by teachers who have been unable to help them contact abstract ideas. Since one of the arguments put forward to support vocational mathematics is that the current curriculum is inappropriate for those who 'fail' GCSE (DfES, 2004; Smith, 2004), it is not going to surprise anyone when the new vocational pathways turn out to attract students from lower-status social groups. Yet it may not be the curriculum which is inappropriate but rather the way it is taught and assessed. So we enter a phase of altering, yet again, the curriculum instead of tackling the underlying systemic features of UK mathematics education. In this new change, we run the risk of

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embedding differentiated expectations, continuing mechanistic, instrumental teaching for lower-attaining students, and reinforcing social underprivilege through grouping practices in schools. In the USA, such distinctions often have a racial dimension and are therefore clearly unacceptable. For example, Powell's project shows that students who had been thought incapable of mathematical reasoning were clearly capable in a supportive environment with high expectations (Powell, 2004).

Needs: Adolescent as Deficient Adult

Who identifies 'needs' for, and of, whom? The rhetoric of 'learning needs' suggests either a diagnosis and treatment model of education, in which some students are seen as deficient and thus have to be normalised or removed from mainstream expectations, or a model of society in which an underprivileged 'lump' is dependent on employment forecasts for its access to training and qualification. In neither case are learners systematically asked to identify their own needs, perhaps because they are considered too ignorant to identify them intelligently, yet several studies show them to be highly aware of how they learn best and the gaps in the teaching they receive (Boaler, 1997; Boaler *et al.*, 2000). Students in the lower achieving school in Boaler's comparative study of two schools longed for mathematics to make sense, and for more time to understand it. Internationally similar results have been found, even in high-achieving countries.

Another approach to identifying 'need' is to offer mathematics which is 'relevant' to adolescents' current social and economic lives, such as the mathematics of the paper-round, or of buying DVDs, or of planning an event. It is often argued that tasks need to be relevant to learners' potential and current interests and current knowledge, within or outside mathematics (Grugnetti and Jaquet, 1996). The nonsense of emphasising solely current interests, an approach which serves to limit the horizons of those whose horizons are already limited by age, ignorance and context, is graphically illustrated in the work of Vithal (2003), who points out that for some pupils the 'relevant' mathematics is that of illegal and dangerous activity on the street. It may be part of the educator's task to extend adolescent experience and invite them to explore a zone of proximal relevance.

Entitlement: Adolescent as Person

It is worth noticing that adolescents whose social antennae are necessarily finely tuned will also conjecture and generalise about the

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groupings in which they are placed for mathematics, and can be insightful about what it means for them to be included or excluded from certain choices. The fact that one top set for mathematics I observed in a London comprehensive consists of 23 white boys, five white girls and two Afro-Carribean boys may be unnoticed by their teachers, but is unlikely to be unnoticed by the students. Students know if they are not being taught mathematics which is hard enough for them to get the best qualifications.

5. Best Teaching

Motivation is needed to engage deeply with mathematics and this is unlikely to be gained through meaningless or trivial questions and the disruption to sense and learning caused by frequent testing. Ahmed (1987) showed that learning skills in isolation from the development of a conceptual structure is demotivating and undermines confidence. The investigative tradition in the UK and elsewhere has focused on the power of mathematics to provide its own intriguing questions. Goldenberg (1996) identified, with colleagues, the 'habits of mind' of mathematicians, such as 'algebraists worry about what is generally true', and devised tasks within mathematics which promote these in school learners. One of Goldenberg's students described what she had been asked to do in such a task as 'realistic' not because it related to an everyday context, but because it made her think in 'real' ways. The underlying belief was that exploring mathematics is in some ways no more difficult than exploring any other ideas, so long as students' self-esteem and existing knowledge are recognised and their natural propensities to compare, categorise, generalise and describe are fully engaged in the task.

It is possible to engage nearly all students in some form of abstract and conceptual understanding, but as long as segregation is supported with arguments about choice, needs and deficit it is likely that significant numbers of learners will find their pathways limited to the utilitarian methods associated with vocational aims, simply because no one has ever managed to find out if they can be taught more sophisticated ideas. Goldenberg, Ahmed, Powell and others show that given commitment and support teachers can make a difference. But even with research results showing that their methods work 'best', individual teachers interpret methods differently (Davis, 1999; Tanner and Jones, 2000). Ahmed and Williams (2004) worked over considerable time in the late 1980s to develop low attaining classes using investigative work, discussions, open questions, choice and problem-solving. The methods developed in their LAMP project

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were so successful that they set up a series of groups nationally to spread the methods to a full range of pupils in participating schools. A central focus of their work was that it was based not on innovative ideas given to teachers but on teachers developing such ideas for themselves within a supportive group, an approach which has recently been replicated in the Improving Attainment in Mathematics Project (Watson *et al.*, 2003). In this project, teachers taught in observably different ways, yet all shared the same underlying principles and goals about the potential achievement of their students, and all were relatively successful. By the end of a second year there was a convergence towards giving students extended time to think about mathematics, more time to work on individual topics, and extended tasks which lasted over several lessons were used more. These moves, like those in LAMP, were the result of teachers sharing practice in a critical, friendly environment, but each teacher's enacted version of these shifts was different.

6. Conclusion

In this paper I have offered a reply to Smith's question about 'best' methods of teaching mathematics. There is no answer to this question because it lacks definition about the nature of mathematics and the goals of education. If, however, we take stated needs from employers and universities as the goals, we find important similarities. The goal of mathematics education should be students who can embark on multi-stage tasks, adapt and create appropriate problem-solving methods, can identify and operate with general principles, can recall and act fluently on whatever the basic mathematical tools are of their chosen field (and know how to identify learn, practice and remember these) and use ICT in an integrated and critical fashion. There are powerful similarities between the emotional state of adolescents, their intellectual state and the kinds of shift which are being made in the secondary mathematics curriculum at the same time.

The kinds of teaching available often fail to match with the adolescent need for support for the process self-actualisation, and for other social, emotional and psychological moves from childhood to adulthood. Teaching can be exploratory and life-developing at all levels of mathematics, but is more often a mixture of offering rules which are hard to follow – a cruel mixture of apparent safety which conceals high risks. It is common to hear teachers of low attaining students say 'they like doing exercises', but then to find that many who are thus 'working quietly' are following unsafe or incorrect

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procedures. When this kind of mixture occurs socially, such as in family life, it can cause serious emotional disturbance. It is hardly surprising then that students in similar classroom circumstances become frustrated and behave ineptly, thus generating a cycle of 'can't be taught so won't teach them'.

If in future students can choose, or are assigned by others, to follow particular curriculum pathways at 14, it seems likely that those who are least confident in every aspect of their lives, least able to operate with the generalities of society, least able to see how they fit, as individuals, into the complex world around them, will be the least likely to opt for pathways which require confidence, independence, risk, abstraction and complexity. Pathways which seem more familiar are more likely to be chosen. Thus a student whose background is rather bookish might be less inclined to choose a practically-based pathway, and a student who spends his non-school life hanging around cars is less likely to choose an abstract pathway.

The aim of good life chances for all is unlikely to be met, therefore, through the imposition of 'best' pedagogies unless it includes an end to early segregation of groups and curriculum limitations. Without these, choice of different pathways at 14 advocated by Smith may only serve to continue the existing relative achievements of certain social groups, and hence contribute to embedded social injustice, while failing to provide the desired educational ends. Different pathways are likely to be manifested in schools through assigning students who are already disenchanted to those routes which are seen to carry less status, less challenge and less personal satisfaction. The chance of real mental empowerment through abstract thought will have been lost due to the failures of an over-prescriptive curriculum, imposed teaching methods and a mechanistic testing regime.

More might be achieved by identifying as 'good' teachers those who make a real difference to the life chances of low attaining students, rather than those who 'fit' current practice guidelines. If this is done, it is likely that their pedagogy will not be uniform and will be hard to describe in replicable terms.

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