**Anne Watson: Comments sent to DfE on the draft Multiplication Tables Check assessment framework in 2018**

**Ambiguity**

It is unclear whether what are to be checked are multiplication tables as traditionally recited, e.g. ‘seven eights are fifty-six’ or multiplication facts such as ‘nine times what are seventy-two?’ or the facts that are most useful in later mathematics, e.g. ‘what numbers multiply together to give 24?’ If the latter two forms are to be tested the timings need to be much more flexible since these require more work than searching verbal recall routines. On the other hand, if only the first kind is being tested this is less useful in later mathematics.

**Scope**

If the argument is that fluent and fast recall is valuable for performing multiplication methods, then only tables up to nine times are necessary and ten times is broadly useful for decimal, standard form and place value calculations. Reducing the demands of the test up to 10x10 would be helpful for teachers and time (see below) and also to convince people of the purpose for testing, i.e. it is only worth testing what is necessary for future learning.

**Misinterpretation**

There is a statement that appears to be untrue. It is false that the cognitive process of recalling different table facts is constant and this statement should not go out under the name of the DfE. Below I summarise the authoritative research on this.

Moreover, this draft document is spuriously supported by a reference to Hatfield et al (2000) which is a book of advice to US teachers and NOT an authoritative text about the cognitive demand of recalling tables. The authors have no record of research or publication in the areas of testing tables knowledge. Moreover the book is not easily available in UK. The DfE should not be using this text to support its statements. More authoritative and available are the following references from cognitive and developmental psychologists and published in refereed scientific journals or edited collections, and these are based on research using scientific quantitative methods.

1. Baroody, A. J. (1994). An evaluation of evidence supporting fact-retrieval models. *Learning and Individual differences*, *6*(1), 1-36.
2. Baroody, A. J. (1985) Mastery of basic number combinations: Internalization of relationships or facts? *Journal for Research in* *Mathematical Education*, *16*, 83-98;
3. Baroody, A. J., & Dowker, A. (Eds.). (2013). *The development of arithmetic concepts and skills: Constructive adaptive expertise*. Routledge
4. Brownell, W. A., & Carper, D. V. (1943). Learning the multiplication combinations. *Duke University. Research Studies in Education.*
5. Brownell, W. A. (1944). Rate, accuracy, and process in learning. *Journal of Educational Psychology*, *35*(6), 321
6. Butterworth, B. (1999) A head for figures. *Science,* 284, 5416, pp.928-929
7. Butterworth, B., Marchesini, N., Girelli, L., & Baroody, A. J. (2003). Basic multiplication combinations: passive storage or dynamic reorganization?. *The development of arithmetic concepts and skills: Constructive adaptive expertise*, 187-201.
8. Delazer, M., & Benke, T. (1997). Arithmetic facts without meaning. *Cortex*, *33*(4), 697-710.
9. Domahs, F., Delazer, M., & Nuerk, H. C. (2006). What makes multiplication facts difficult: Problem size or neighborhood consistency?. *Experimental Psychology*, *53*(4), 275-282.
10. Krutetski, V. A. (1976). *The psychology of mathematical abilities in school children*. Chicago: University of Chicago Press.

**Issues arising from the literature**

Many children do not rely on verbal rote memory alone for their multiplication facts because their experience of numbers, and of reciting tables, has visual and structural components as well as, or instead of, only verbal rote. This is because the mind tries to make sense of what is said, and also looks for patterns and structures that make memory easier and more meaningful. This means that time taken to recall facts varies according to the way a particular fact has been organised in relation to other facts. For example, if a child has a rhythmic component to their memory of a times table, they may need to search through the recitation from the start to find the desired fact. Krutetski (see paper x) who studied children who were at Soviet schools for gifted mathematicians, found that memory for mathematical facts was not a common trait among these students – the strongest in mathematics - but memory for mathematical structure was.

To quote from Brian Butterworth in papers vi and vii.

“Even the simplest arithmetic is more than just fact retrieval. Indeed, there is evidence that children learning multiplication tables are not simply passive recorders of the verbal form, but reorganize the facts to make them easier to retrieve—for example, by using a single preferred form for commuted pairs such as 6 × 2 and 2 × 6”. Butterworth’s research leads to the conclusion that “the child learning multiplication facts is not passive, simply building associative connections between problems and solutions as they are experienced in recitation or in problem presentation. Rather, the facts in memory seem to be reorganised in a principled way that takes account of a growing understanding of the commutativity, and perhaps other properties of multiplication”.

Paper vii includes the information that 3 seconds is not enough for many ages and questions since this is less than the mean time for the equivalent of our year 4 students in their tests (the paper is about more than this, but the results table gives this information). To expect UK students to have significantly faster reaction times overall needs to be tested. I note that time per question is going to be 6 seconds to allow for reading time. Where is the properly researched evidence to support this assumption?

Whereas there is evidence to show that multiplication facts CAN be stored solely in those areas of the brain associated with language, it is not the case that children are only going to be verbally ‘trained’ to do multiplication. On the contrary, the aims of the curriculum make it clear that children are also going to be taught to understand the process, to become familiar with number structure, and to develop strategies – relevant to this matter are mental strategies, such as deriving 7x8 from knowledge of square numbers, 49 or 64 (a strategy that many adults have used to aid rapid and accurate recall). Butterworth has researched the strategy of reversal using commutativity to change a high value, late-learned fact such as three eights, into a low value, early-learned fact such as ‘eight threes’. Children’s learning will not have been purely verbal, indeed it would be pointless to do so since use of tables needs to be associated with the meaning of the numbers and the operation for them to be useful. The main researcher of this area is Delazer and her team, (papers viii and ix) but it is worth pointing out that some of her research was on one brain damaged subject, and other research relates to adults being taught some double-digit multiplication facts an average of 15 minutes per fact before testing. Those who had this preparation did better than those who had strategic preparation, but the time-cost for doing this with school children for all the table facts makes no sense, given that what would be lost would be the meaning and understanding and usefulness for future adaptive reasoning.

**To conclude**

While 6 seconds may well be enough for many facts (those more often used or accessible by a range of ways in which memory can be organised), there needs to be proper research done to verify this given that recall is NOT going to solely based on verbal rote learning. Note particularly in Butterworth’s paper that Chinese children only have to learn half the facts our children are expected to learn, since they then adapt to the commutative form that is easiest to remember. Our children will not have that advantage since they are expected to learn by rote both forms of every table fact.

The alternative is to keep children away from making any personal meaning of multiplication facts and get them to recite only – but this would be contrary to the overall aims of the NC and quite barmy in terms of their mathematical development.