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MATHS 14-19: ITS NATURE, SIGNIFICANCE, CONCEPTS AND MODES OF ENGAGEMENT

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The nature of mathematics and its modes of engagement

The National Curriculum for Mathematics declines to state aims, but describes the subject as follows:

> Mathematics equips pupils with a uniquely powerful set of tools to understand and change the world. These tools include logical reasoning, problem-solving skills, and the ability to think in abstract ways. Mathematics in important in everyday life, many forms of employment, science and technology, medicine, the economy, the environment and development, and in public decision-making, Different cultures have contributed to the development and application of mathematics, Today, the subject transcends cultural boundaries, and its importance is universally recognised. Mathematics is a creative discipline. It can stimulate moments of pleasure and wonder when a pupil solves a problem for the first time, discovers a more elegant solution to that problem, or suddenly sees hidden connections.
> (DfEE, 1999, p.14)

While this is true, it is too vague to be helpful educationally, but the common international view is that learning mathematics involves:

- knowledge of concepts, techniques, notations and relationships;
- recognising them in familiar and unfamiliar forms;
- recalling facts, names, procedures;
- reasoning about concepts, techniques, notations and relationships;
- using procedures fluently and accurately;
- the ability to shift between methods and representations;

- applying knowledge to solve problems, possibly transforming it to do so;
- creating generalisations, abstractions, images and methods (Niss, 1996).

All this takes place in numerical, spatial, statistical and abstract domains. Mathematics is unique in that its modes of enquiry constitute the major part of its subject matter. For example: its standard of justification is proof, and proving is 'doing mathematics'; calculation is both a mode of enquiry and a body of techniques which constitute some conceptual content.

The nature of mathematics and its significance

The aims of a mathematics education would be to identify a subset of the possibilities presented by the subject which provides an amalgam of:

- utilitarian: learning a 'useful' collection of skills and procedures (social/ economic-centred aims)
- academic/technical: providing a basis for higher education (subject-centred aims)
- humanistic/critical: developing application and problem-solving in order to participate fully and in an informed manner, in society (person-centred aims)

Smith (2004) identified the importance of:

- Mathematics for its own sake: universal language and intellectual tool-kit for abstraction, generalisation and synthesis; logical reasoning; analytical problem-solving; it 'trains the mind';
- Mathematics for the knowledge economy: science, technology, engineering, finance;
- Mathematics for the workplace: mathematical literacy which encompasses: interpretation and use of different representations of data; data-entry and monitoring; related communication skills; recognition of errors and anomalies; knowledge of what, how, and when to calculate; use of relevant degrees of accuracy and plausibility;
- Mathematics for the citizen: access to labour market and general social and political inclusion.

Contribution to young people's development

From the above, mathematics can be seen to contribute significantly to a learner's future **employment and educational skills and prospects, and full and critical inclusion in society**. During the 14-19 phase there are further contributions it could make, as follows.

Mathematics also has the potential to contribute to the development of **analytical and reasoning skills**, and through these to personal and social empowerment.

It is well-known that difficulties with mathematics can lead to intense emotional reactions which block future learning, and that this is often embedded during early adolescence and before, hence it is also true that overcoming these in the 14-19 phase can contribute positively to **self-esteem**.

While the necessary move from naïve and ad hoc understandings, and procedural approaches, to abstract and structural understandings involves many obstacles, this shift also moves learners from dependence on teachers and textbooks for validation to being able to provide their own validation and hence to a **sense of power**. If this shift is carefully handled by teachers, it can provide scaffolding for similar aspects of adolescent development outside maths. Mathematics can quintessentially offer **intellectual choice** and absence of adult authority.

What is working against this contribution?

Teaching methods

There is a strong match between outcomes desired by employers and exploratory, collaborative, teaching methods which also encourage development of appropriate and fluent skills. However, recent Ofsted reports indicate that uncritical adoption of teaching methods advocated at Key Stage 3 has led to teachers offering too limited a time to learn, moving through the curriculum too quickly and with too frequent changes of topic for lasting learning to take place, especially for the lower attaining students. The programmes of study offered to students can be too limited, too early, thus restricting opportunities to learn (Ofsted, 2000, p.12).

Testing

Submissions to the Smith Report were overwhelmingly of the view that frequent testing led to a fragmented approach to the subject and underdevelopment of flexibility and problem-solving skills, both for university entrants and students entering employment. Thus the enacted curriculum is relevant for nineteenth and early twentieth century purposes, rather than current needs, values and technological possibilities.

Teacher supply and professional development

Shortage of qualified mathematics teachers in secondary schools is also an obstacle, and there tends to be deployment of less well qualified staff to teach lower attaining groups. Younger teachers who have only ever known the current testing regime may not know how to teach mathematics in deeper, less test-orientated ways.

ICT

ICT use is not integrated into the teaching of mathematics in the ways in which it acts in the workplace, in academia, and in the world. Ways of working on mathematics in school are old-fashioned in comparison to all adult engagement with maths.

Political targets

There is remarkable professional agreement among mathematics educators, universities and employers about the need for complex mathematics teaching which achieves these multiple purposes in an integrated way. Government efforts to improve attainment in mathematics, which had initial intentions that largely matched those of employers and universities, have effectively prescribed ways of teaching a very full curriculum in ways which work **against** such long-term education into ways of acting mathematically in and with the world for many students.

Setting

Setting students according to prior attainment leads to lower attainment for a significant number of students than would be achieved in mixed ability groups, and presents them with a more limited curriculum and more proscribed ways of working. The different treatment given to lower groups is not designed to raise them to the level of other groups, even though the justification for setting is often in terms of 'needs', which implies enabling similar achievement through different teaching methods.

Work-based learning/vocational courses:

A substantial research study by Hoyles et al (2002) defines 'mathematical literacy' as encompassing the mathematical needs in the workplace. This includes:

- interpretation and use of different representations of data;
- data-entry and monitoring;
- related communication skills;
- recognition of errors and anomalies;
- knowledge of what, how, and when to calculate;
- use of relevant degrees of accuracy and plausibility.

The relationship between learning mathematics in a formal educational environment and learning it in the workplace situation is the focus of a great deal of research, as is the more generic problem of transfer. It is generally agreed that formal skills learnt in school are not always relevant, nor do they transfer to other situations. This does not mean that other skills would be more relevant and transfer better; it means that knowledge and practice are essentially situated and more attention needs to be given to the practices within which knowledge is created. The more a classroom reflects the ways of working and ways of being (including tool-use, power relations, forms of communication and so on) which are relevant in the workplace, or in higher education, the more 'relevant' the learning will be.

How far my views are shared

What I have reported above is fairly mainstream, apart from the section on adolescent emotion, empowerment and mathematics which I have not heard elsewhere. Nor have I read anything explicit about the common ground held by employers and university mathematicians in terms of the 'output' of school mathematics, apart from in the Cockcroft Report of 1982 and the Smith Report submissions.

Main areas of debate are:

Setting: Despite the overwhelming research evidence of the negative effects of setting on average and below average attaining students, this continues to be almost universal practice and government policy. Very few countries set for mathematics. Our official stand is that England offers the same curriculum to all taught in different ways. This is just not true in practice, or even in theory at 14+.

Choice of pathways at 14 plus: The proposal that there should be choices of academic or vocational pathways at 14+ is contentious, depending as it does on the cumulative effects of setting policies which relegate those from less advantaged social groups into educational pathways which are designed to lead to limited achievement. If the pathways are designed to be equivalent, experience in other countries suggests that choices still cleave along class lines.

Maths for all: All European countries and other developed countries, and some developing countries, expect all students to study mathematics until school-leaving level. In this country there are strident voices suggesting that it could become optional at 14 (see Bramall and White, 2004).

Vocational pathways: The nature of new vocational pathways is currently under discussion, and I expect the debate to hinge around their intellectual equivalence, or otherwise, to other routes. I shall know more after March 3rd.

Key references:

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