Dance and mathematics: power of novelty in the teaching of mathematics

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Abstract

In this paper I illustrate how kinaesthetic experiences of mathematics can be used in teaching to promote engagement and learning. I briefly explore the possibilities and limitations of such an approach, and of 'novel' approaches in general.

Introduction

Educational institutions searching for 'quick fix' solutions to underachievement may be tempted to adopt one of the many 'theories' currently offered which advise teaching different students in different ways according to their 'preferred learning styles'. For example, in some schools students are tested to find out if they are visual, aural or kinaesthetic learners and teachers then advised to teach them accordingly. Gardner (1993), who saw his list of learning styles as a conjectural categorisation, not as a recipe, included 'musical' and 'logico-mathematical' as possible preferences too. As well as the obvious problems in organising radically different treatments in whole class settings, or the social dangers of segregating students identified as 'different', teachers may also be puzzled about how to teach mathematics in kinaesthetic or musical ways, when 'logico-mathematical' seems the obvious way to proceed.

In this theoretical paper I describe some ways in which links between mathematics and dance have been, and can be, exploited educationally (Tytherleigh & Watson, 1987). I take 'dance' to be deliberate, planned movement for aesthetic and purpose and/or to express or convey meaning. My data comes from personal experience of working with school students, workshops with teachers, and the reported experiences of teachers on the tasks described below.

There are at least four aspects of mathematics which can be related to dance: spatial exploration, rhythm, structure and symbolisation.

Spatial exploration

A child's first experience of space is three-dimensional and this is explored through movement both inside (e.g. being in the womb; hiding under a table) and outside (e.g. playing with cubes). Thus we know that most people can learn about shape by physically interacting with it (Tahta, 1999). Indeed some psychologists claim that we learn emotionally and conceptually by interacting with the spatial environment (Winnicott, 1955; Maturana & Varela, 1988; Vygotsky, 1994), while Lakoff and Johnson claim that space provides central metaphors for our living and learning (1980 p.29). Laban, who created an educational form of dance, used interacting icosahedra to represent directions and qualities of movement which could be used for different physical and emotional purposes, recognising that three-dimensional geometrical forms provided a way to organise and represent space (Laban, 1966). Trisha Brown, a choreographer, used this explicitly in asking dancers to imagine themselves inside a cube and touching various features of it, vertices, midpoints and so on, with various limbs, knees, elbows and so on

(Kino, 1999). Using physical imagination to explore shapes from the inside can be used for geometrical education with students; one could prompt exploration of the curvature of an imaginary sphere, or trace out the arcs of an octahedron, or get a feel for axes and planes of symmetry. Beyond geometry, one could experience the continuity and differentiability of surfaces.

Any part of the body can trace out a circle and feel its centre, the constancy of its radius, and the plane in which it lies. That sense of circularity is used by many teachers to convince learners of the sum of external angles of any polygon. Students walk once along all the edges of an imaginary polygon on the floor, being aware of turning the corners, they then know that they have turned through 360 degrees, and also know where the external angles are and what they look like, when they are later looking at diagrams. Bruner's theory of instruction offers shifts between enacted, iconic and symbolic modes of representation as necessary for conceptual learning (Bruner, 1966). Usually these are offered as occurring in a cycle, yet researchers in Hawaii have found that young learners respond well to experiences which nest these together (Dougherty, 2003). Similarly, neuroscientists confirm the intuitively obvious fact that learners who have been offered several modes of representation generate more brain activity as they try to reconcile these stimuli. Papert's understanding of the connection between motion and logical experience, normally expressed through turtle geometry, can link kinaesthetic, visual and symbolic to the logico-mathematical senses, if indeed these are separable as Gardner implies (Papert, 1980). In the 'walking round a polygon' task there is a two-way link which teachers exploit between actual physical movement, (enacted) movement, and representations (icons) which are memorable and meaningful. Thus a connection between action, memory and diagram is created. A diagram is not just a representation of an abstract object; for the students it *feels* like a map of their own movement.

The drawing of a circle on the floor can be done either through motion of a whole body, or through motion of one part of a body, or through the static imagination, or by imagining the movement of someone or something else, or by actually drawing it so it can be seen visually, or by representing it on a diagram, or by projecting it from somewhere else. For example, the challenge to draw a circle on the floor with your left foot as the centre is fairly easy, but to have your elbow at the centre requires some thought about rotating bodies. Even harder, the exercise of imagining you have a laser light shining from your finger and have to draw a circle from a standing position is mathematically non-trivial since you have to 'act out' a conic section. Trying to translate this experience onto paper is also hard, but students who try this can be seen to be waggling their fingers as they draw, or getting up with the piece of paper and trying to re-enact the 'laser light' play using the diagram they have just drawn.

These activities combine action, imagery and thought in complex ways. There is not simply a unidirectional flow from action to thought, nor from thought to action, but a messy cycling between modes. Sometimes the necessities of action lead to realisation and reasoning ('it *must* be so because …') and at other times thought has to guide action ('I *have* to do this because …'). But these flows are hard to separate in the nexus of solving the current problem, and may not be separable. When students engage with such tasks, the role of speech is also multi-dimensional. A very common phenomenon is for a student to find an individual space, explore the task physically, mutter as if words capture a finding, or as if words provide an alternative form of reasoning which has to be uttered to separate it from physical reasoning, and later discuss their 'results' with others using physical movement to illustrate the words. Thus the balance between physical and verbal forms of reasoning and expression varies at different stages of the work.

This kind of work suggests not only that physical exploration of shape is valuable learning experience, but also that appealing to physicality, that is physical imagery and physical memory, is a useful teaching device. All a teacher is doing when introducing movement to explore shape is making overt use of what is already around in the way students experience the world.

Rhythm

Kinaesthetic and musical sensitivities join together in the rhythms of dance. Many people need to respond physically to certain rhythms, either feeling them resonate within or by toe-tapping or getting up and dancing. How can this be seen mathematically? At a very elementary level, there are links which can be made between rote learning and rhythm, such as choreographing the eleven times table to emphasise the 'tumty-tums'. Rather less obviously teachers exploit classical rhythms to develop a sense of fractions, as musical notation does in time signatures and note values. The fraction family of: ½, ¼, 1/8, 1/16, can be added and subtracted in common time; the family: ½,1/3,1/6, and so on can be manipulated in 6/8 time. The added feature of dance can be used to show students that they know these relationships already through their movement, so that fractions express what their bodies can already do. Teachers who do this report that students are quite excited to know that their rhythms can relate to mathematical symbols.

Use of rhythm has connections with the experience of chanting. Once one is 'caught up' in the rhythm one pays attention to pattern rather than meaning, 'living' in the structure; when the rhythm breaks down one rapidly returns to conscious thought about meaning. An exercise which demonstrates this is to chant the successive results of subtracting $1\frac{1}{10}$ starting with 101. Typically, chanters find their attention varying between individual digits, pattern, subtraction, and what others are doing (especially when they get lost).

Structure

Abstract representations of structure, such as permutations, combinations, graph theory and groups, are manifested in many traditional dances. English country dances, for example, usually include actions of combination and their inverses; provide patterns of interaction which ensure that every possible link in a digraph is exploited; and end up back in a starting position after everyone has had an equal role (Playford, 1651). Thus they show the characteristics of a group and, like traditional bell-ringing and braiding, provide a cultural manifestation of a structure which can be expressed through symbols (Roaf & White, 2003). The experience of identifying what is essentially common about a set of traditional dances, claiming this 'truth' to be a definition, using the definition to create a new dance and finding a way to represent it symbolically, could be called an experience of mathematising. In particular, the need to distinguish between dancer, actions and relationships between actions replicates the distinctions which have to be made in abstract algebra, yet personal involvement in the action can make these distinctions obvious, whereas students of abstract algebra typically find them tricky. This route of access to algebra has been used successfully with teachers who are improving their own understanding of the breadth of mathematics in order to improve their teaching at lower secondary or upper primary levels.

Symbolisation

In each of the above examples of the potential use of dance to motivate mathematical ideas is about a relationship between movement, mind and memory which develops through experience and the senses. However, Laban used the icosahedron not just to model space, but also to record movement in a notation. Dancers and choreographers seek notations which can convey the complexities of dance, so that its communication and preservation do not depend on a continuous line of teachers. There are many routes by which notation has been sought. In square dancing, once floor shape is established, movement is conveyed by words, some of which have to be translated, such as 'dozy-doh' for back-to-back and 'strip-the-willow' for something really complicated. Ballroom dancing is conveyed by a floor diagram showing foot positions with added nuances to indicate movement type. The 18th century dancemasters had floor diagrams which included signs like musical notes to represent rhythm and movement type (Feuillet, 1700; Tomlinson, 1735; Tufte, 1990, p.114-119). These combine iconic and symbolic modes of representation, in that the drawn symbols consist of a scale diagram of positions, with added squiggles which have to be translated to have meaning. In Laban's notation, a higher level of symbolisation is used, there is nothing left of the icosahedron itself. Thus dance can be used to demonstrate how symbolisation arises from a need to record the seemingly unrecordable. Once symbols exist they can be used to communicate to others who hold the same codes, and also to create new objects through the internal rules of manipulation for those symbols. In this case the new objects are dances, but there is of course a similarity with music here (Fauvel, Flood & Wilson, 2003).

Learning from novel contexts

In all the above suggestions, what is offered are NOT 'kinaesthetic ways to learn mathematics'. Instead, they are examples of how the human mind is able to operate in physical – mental, concrete-abstract, iconic – symbolic, modes simultaneously, not cyclically, if the context of activity itself demands engagement of mind and body. Each of these ideas has been used by teachers who recognise that the complexities of dance (or music, or crafts) can be made explicit and used to show firstly the power of mathematics and secondly the powers of the learners themselves to handle such situations.

However, such activities can also be a distraction from mathematics if they are not integrated into the learners' overall mathematical experience. At worst, students only remember the dancing. In attempts to help learners become more interested in mathematics, students can be exposed to exciting ideas in mathematics, can glimpse something which makes them excited, and then return to their normal classroom lives and find nothing which relates to what they have recently enjoyed. Often a 'popular' approach offers mathematical ideas which are too hard for school students to comprehend and the potential of such novelties to engage interest is lost because learners cannot connect it to their school lessons. It is too much to expect learners to work hard on their exercises today with the hope of becoming famous scientists in twenty years' time! In order for such experiences to have more than novelty and motivational value, they need to relate closely to classrooms in terms of what it means to do mathematics. Students need to be able to pick up and use some of the ideas presented in ‘novel’ contexts in their normal lessons, as habit. Innovation, at its best, makes a difference to how learners think in *all* their experiences of mathematics, not just in the innovative mode.

I am not suggesting that dance has the key to this, indeed it would be very easy for all the potential offered above to dissipate by emphasising only cooperation, rehearsal, product and performance (much as many mathematics lessons do!). Also, not many mathematics teachers would feel confident about such an approach. Rather, I am offering dance as one of several examples of activity which can reveal and enhance learners’ abilities to do some of the things which are useful in mathematics, and which, if used soon after in a more purely mathematical context, might lead to a deeper understanding of how to engage in learning. Ways to engage students’ minds through aesthetic and physical senses, and to relate these to iconic and symbolic modes of representation, in the learning of mathematics might become easier to find through considering contexts and tasks in which these connections arise naturally.

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