WHY PROBLEM SOLVING

Learning mathematics by reading a text is like learning a game from the book of rules. Written solutions to problems are like the sports pages of a newspaper just scores and commentary.

Learning mathematics by listening to a lecture is like learning a game by watching professionals play.

BUT
Problem Solving
is
PARTICIPATION

Which
is most accurate:

- I solved the problem,
- The solution came to me,
- I participated in the problem solving itself,

?

"You've got to solve problems in order to practice routine skills"

"A problem presents itself - it must be solved. There's a nagging curiosity that won't go away".

"The joy of being present at the solution is worth all the frustration that precedes it"

Often people don't try to solve problems because they're afraid to fail. The fear comes because they measure themselves by success. What is important is to participate in a problem - solutions or not!

The problems in this book are useless once the solution is revealed. The whole point is the experience leading up to the moment of solution.

A problem isn't solved until you've successfully explained the solution to someone else.

Lynne

CHEZ ANGELIQUE

John Jaworski
John Mason
Alan Slomson
et al.

The Bumper Late-night Problem Book.

THIS BOOK

is a private publication; it has no connection with the Open University Press, nor with the Mathematics Faculty, other than in the fact that without the practice, philosophy and support of the Open University, none of this would ever have happened.

is not a profit-making venture; any proceeds will be used to finance similar publications in the future, to support the promotion of recreational mathematics in the Open University and to add to the funds available to the Open University Student Hardship Fund.

HOW TO USE THIS BOOK:

This is a book about problems, and about solving them. The problems are only rarely original. Some are of painful antiquity. We are in no sense authors, merely editors.

The book has come about because of something special that happened at some of the Open University's 1975 residential Summer Schools at Stirling University. There seems to be no good reason why the ideas behind this book should have shown fruit when they did. What happened could have happened at any time, or in any place.

What happened was CHEZ ANGELIQUE (at an earlier time, Club Kassab); This was simply a mathematical nightclub - a cross between a cabaret-act and a folk-club perhaps. Tutors and students alike got up from the floor to present their puzzles. The range was enormous: mathematical puns, logic 'quickies', problems without solution, research problems from the frontiers of mathematics.....

Best of all was the enthusiasm - students left the bars early to be sure of good seats. The room was never less than full, and nightly people were turned away. People fought to get in, and stood at the back of crowds to hear the words without the actions.

In the last week of all, many students who had been invigorated (or intoxicated?) by recreational mathematics begged us to put the problems into some more permanent form. This we have tried to do in the present volume

We should have liked to have forced readers to try to solve (or to resolve) each puzzle for himself. The simplest way to this end would have been to have omitted the answers! We recognise that this might not be to everyone's taste, however. But to try to encourage you to see these problems as a challenge, we have adopted certain conventions of typography and layout, listed over the page.

Problems and puzzles are presented, in a continuous seq
beginning on page 5, in this type face.

Answers and solutions are to be found following page 42, ordered as the problems themselves, and in this type face. A page-number reference to the solution of a puzzle is to be found in the CONTENTS.

A number of articles, descriptive pieces &c are to be found, interspersed with the puzzles. They appear in this distinctive type face, in order that they may not be confused with the problems.

Most puzzles are accompanied by a short comments; frequently the comment includes a hint or hints, either explicitly or disguised in a horrible pun. The editors take some pride in the grotesque nature of these puns.

Contents

VOWEL !

On these four cards

2	B	A	3

I guarantee that you will find one side printed with a letter, and the other with a number. Imagine, if you wish, that this fact has been independently verified by someone that you trust.

I now make a claim - which may be either TRUE or FALSE. I claim that on the back of a VOWEL, you will always find an EVEN number.

<u>PROBLEM:</u> <u>EXACTLY WHICH OF THESE CARDS MUST YOU TURN OVER AND EXAMINE IN ORDER TO VERIFY OR DISPROVE MY CLAIM?</u>

This is best done with a group of people. First find out how many cards each person wishes to turn over. Then get the most vociferous pair with different ideas to argue it out! But don't be sure that YOU are right - there is more than one answer!
If VOWEL! has given you a taste for this style of 'experiment', try this:-

SQUARES & CIRCLES

I have here four cards, each with a symbol on as shown in the diagram below. I also have a fifth card, on which one of the four symbols is repeated.

■	●	○	□
a black square	a black circle	a white circle	a white square

The fifth card is hidden. So that you can relieve yourself of the nagging doubt about what is on the fifth card, I am willing to answe[r] questions of the form "Does it agree (in colour, shape or both) with the black square (say)?" Unfortunately, there is only time for one question - let us suppose that you have indeed asked "Does it agree with the black square?". My answer is "Yes"

PROBLEM: <u>CAN YOU PREDICT, ON THE INFORMATION SO FAR, ANY CARD TO WHICH I WILL ANSWER NO?</u>

With this sort of problem, there is little value in immediately analyzing the situation precisely. What is interesting is analyzing the difference between people who reason intuitively, and those who reason deductively

POLITICIAN'S PROMISES

Here are 5 numbered statements:-

1: One of these statements is false *(i.e: exactly one)*
2: Two of these statements are false
3: Three of these statements are false
4: Four of these statements are false
5: Five of these statements are false

PROBLEM: <u>WHICH OF THE STATEMENTS ARE TRUE?</u>

This problem can be a useful insight into the way in which a correct statement of a problem can almost write the solution for you. Looking at things from the other way round is almost always a useful mathematical technique.

HOW TO GET EVERYTHING WRONG - OR: YOU CAN CONFUSE ALL OF THE PEOPLE SOME OF THE TIME!

I was going to invite ten people to a party (a rather quiet affair). I wrote individual invitations, then all the envelopes. I then poured out a quick drink for myself, and went out to buy the stamps, stopping off on the way in the pub. On the way back, I met a friend, and we had a quick pint or two together.....

Next morning, I remembered, through the painful haze, that on coming home I had put the invitations into the envelopes and posted them. However, such was my state of mind that I must have put them in quite randomly, though I'm pretty sure that I would not have put more than one into any envelope.

PROBLEM: <u>WHAT IS THE PROBABILITY THAT ALL THE INVITATIONS WENT INTO THE WRONG ENVELOPES?</u>

A good friend of mine entertains on a more lavish scale; he invited a million people to his party, and the same misfortune befell him.

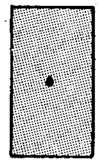
PROBLEM: <u>WHAT IS THE PROBABILITY IN HIS CASE? DOES IT DIFFER SIGNIFICANTLY FROM MINE?</u>

Before trying these problems, make intuitive guesses at the answers; it's always instructive to test the accuracy of one's intuition. Then try to find some sort of recurrence formula, which expresses the probability for n *envelopes in terms of the probabilities for less than* n. *It's a good idea to calculate explicitly the probabilities in the cases of two, three, four and perhaps five envelopes.*

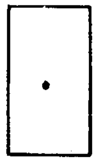
Use this recurrence formula to calculate the n=6 case to 4 decimal places. You should now be in a position to make an educated guess at the general solution of the problem. However, you will probably not be able to <u>prove</u> that your guess is correct, unless you use the following set-theoretic principle (called the Inclusion-Exclusion Principle):
Suppose you have n *finite sets of objects:* $S_1\ S_2\ \ldots\ S_n$, *but these sets overlap. You know exactly how many objects there are in each set, and how many there are in each intersection. (That is, you know how many objects there are in:* $S_1 \cap S_2$ *; in* $S_1 \cap S_2 \cap S_3$ *; in* $S_2 \cap S_3 \cap S_4$ *; etc). Then we have the following formula for the number of sets in the union:* $S_1 \cup S_2 \cup S_3 \cup \ldots \cup S_n$

$$N(S_1 \cup S_2 \cup \ldots\ldots \cup S_n) = \sum_{i=1}^{n} N(S_i) - \sum_{\text{all pairs}} N(S_i \cap S_j) + \sum_{\text{all triples}} N(S_i \cap S_j \cap S_k) \; \vdots \; +(-1)^{n-1} . N(S_1 \cap S_2 \cap \cdots \cap S_n)$$

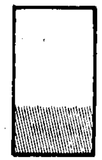
DOWNING ONE'S BEER



Over here, on the right, you can see a very welcome sight: a full beer can. By a fairly reasonable appeal to the symmetry of the situation, the centre of gravity of the can plus the beer is in the centre, where it's marked.



Now take a look at this can - a beerless can - to which we can apply the same arguments about symmetry, and conclude that the centre of gravity is again in the centre.



Finally, look at the third can. Our intuition suggests (correctly) that the centre of gravity is somewhere below the middle. Exactly where, we're not too sure.

The situation suggests that the height of the CG is a continuous function of the level of the beer - a small change in the level of the beer results in a small change in the height of the CG. There is thus a minimum height achieved somwhere between a beer-full and a beer-less can.

<u>PROBLEM:</u> <u>WHEN IS THE CENTRE OF GRAVITY AT ITS LOWEST POINT?</u>

This is a rather nice problem, not least because most people will never have paused to consider what happens to the centre-of-gravity of the beer can that they're emptying (and a good job too, we say!).

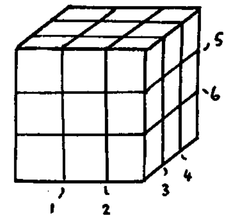
It seems only reasonable to assume a uniform beer-can (even uniform beer) and to neglect the hole in the top of the can.

This is the type of problem where a lack of information is suggestive; no dimensions are given for the can, nor yet relative densities of can, beer and air. We might guess that the solution can be expressed in a form that is independent of any of these. This by itself almost leads us to the solution.

However, the best hint is to try looking at things from another point of view; we could ask ourselves what properties of the CG could be [illegible]

CHEESE SLICES

This is something of a preamble to the problem of the Hungry Mouse. Imagine if you will, a cube made out of cheese, say 3"x3"x3". By making six cuts, as shown below, we can clearly cut the cube into 27 1"x1"x1" cubes.



However, this assumes that we shall keep any sliced-up portions in the same position relative to the large cube. By rearranging the pieces of cheese between cuts, it seems likely that we shall be able to reduce the number of cuts necessary.

<u>PROBLEM:</u> <u>WHAT IS THE SMALLEST NUMBER OF CUTS NECESSARY TO DIVIDE THE CUBE INTO 27 SMALLER CUBES?</u>

THE HUNGRY MOUSE

Given the sliced cube described in the previous problem, all (all!) that you have to do is to show how a mouse could eat his way from <u>any</u> of the outside smaller cubes, consuming a complete small cube at a time, passing from cube to cube across a common face (i.e: horizontally in either direction, or vertically, but not diagonally) to consume the whole cube, <u>finishing with the central cube.</u>

<u>PROBLEM:</u> **<u>CAN YOU SHOW THE MOUSE ANY PATH TO SATISFY THE ABOVE CONDITIONS?</u>**

The connection between these problems lies only in their use of a cube of cheese. They call for different methods of solution. Trial-and-error will convince you in both cases that the solutions, if any, aren't too easy to spot.

Any hint for the Cheese Slices problem is likely to give the game away, and the solution is a nice piece of lateral thinking. Concentrate on the fact that knife-blades don't have right-angled bends in them!

A slightly tantalising hint for the Hungry Mouse would be to ask whether he likes Blue Stilton or White Stilton. On the other hand, you might spend as much time deciphering the hint as you would have spent solving the problem.

GIVE ME A NUMBER!

In fact, give me a three-digit number, and don't tell me what it is.

Right? Well let's make it a little more difficult for me. Turn your 3-digit number into a 6-digit one, by just repeating the same three digits (like either of the two examples on the right).

Now sir - and you madam - please divide your respective 6-digit numbers by SEVEN. <u>There will be NO remainder</u>.

What? Not a single remainder out of all 120 people at the Summer School. Let's try again. All of you divide your quotient by the next prime - ELEVEN. <u>**There will be** NO remainder</u>.

Double or quits, I hear you cry? Very well - divide that last quotient by THIRTEEN. <u>There will be NO remainder</u>.

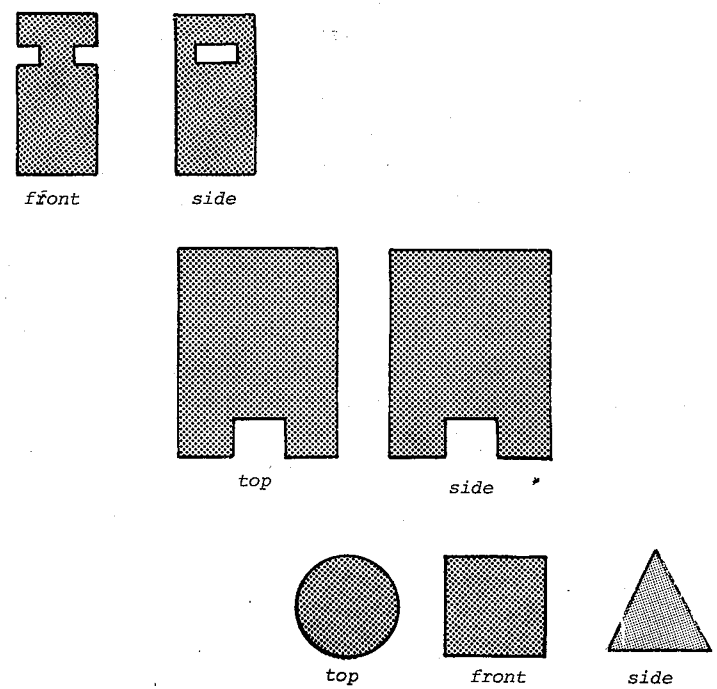
123	165
123123	165165
123123/7 = 17589	165165/7 = 23595
17589/11 = 1599	23595/11 = 2145
1599/13 = 123	2145/13 = 165

<u>**PROBLEM**</u>: <u>WHY ON EARTH NOT?</u>

*This is one of my favourite 'quickie' problems. **Favourite because it can be performed** with an audience of any size - the larger the better - with a very **convincing line of patter.** And also because of one or two features that will **be better discussed** after you've worked out how it all happens.*

NEW VIEW

On going back to my room after Chez Angelique I found myself tripping over objects in the dark. All I managed to see was a shadow as I bumped into it. After the collision I would get a second view of it. Your problem is to reconstruct the object.

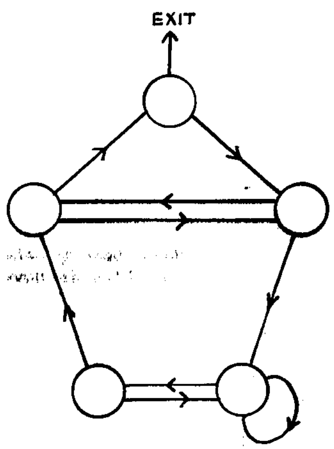


*If you haven't seen these **before**, they can be very frustrating. Once you look at the solution however, the problem is never the same again, unlike others where you can forget the solution!*

THE AMAZING MANSION MYSTERY.

I want you to help rescue John Mason. He is trapped in a large mansion built by an eccentric millionaire (who acquired his fortune touring the country auctioning pound notes). All the rooms in the mansion are circular. They are connected by passages and since the doors between the rooms and the passages have handles on one side only, it is possible to walk down each passage from one room to another in one direction only. Any number of passages can lead into a room but only two passages lead out of each room. Thus when you enter a room and look round it clockwise, one of the exit doors appears to the left of the other. (Of course, which door is the left hand exit may depend on which door you enter the room by). The mansion has only one exit but there is at least one route to the exit from each room.
At present, John is in one of the passages about to enter a room. He has managed to send me an S.O.S. message on a short wave radio set, but now the transmitter has given up and he can only receive messages. In any case he is so befuddled by rushing around trying to escape that he is no longer capable of recognising whether he has entered a given room more than once, and the only instructions he is capable of understanding are those telling him whether to take the left or right hand exits from rooms as he comes to them.

Fortunately the millionaire consulted me when he was building his mansion and so somewhere in my desk is the plan. (An example of what it might look like is shown alongside.)



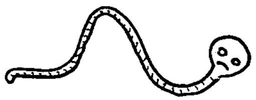
PROBLEM: <u>CAN I SEND JOHN A FINITE SEQUENCE OF INSTRUCTIONS SUCH AS "LEFT,LEFT,RIGHT,LEFT" THAT WILL GUARANTEE TO GET HIM OUT OF THE MANSION WHEREVER HE HAPPENS TO BE IN IT?</u>

So I am looking for a method that would enable me to construct such a universal escape instruction I_1, I_2.........,I_k where each 'I' is either "left" or "right" from any possible plan of a mansion.
(Try first to get a universal escape instruction for the plan shown, then look for a general method).

I have bad news for anyone who managed to find such a method. On looking in my desk, I find that the mice have been at the plans and they are now illegible. Clearly, not knowing the plan of the mansion, I can no longer find a finite sequence of instructions which will guarantee escape.(Since given any such finite sequence, it is easy to construct a plan of a mansion and a starting point such that following this sequence will merely take you back to where you started from.)
However, the problem now is: <u>CAN I GENERATE A POTENTIALLY INFINITE SEQUENCE OF INSTRUCTIONS SUCH THAT WHICHEVER MANSION JOHN IS IN AND WHEREVER HE IS IN IT, BY FOLLOWING THESE INSTRUCTIONS HE IS BOUND TO REACH THE EXIT EVENTUALLY?</u>

THE WORM ON THE ROPE.

You may be forgiven for not having spotted the rope stretching between the University of Stirling and the Wallace Monument, 1km away. Consider a mathematical worm (that is to say, a point worm, of negligible mass and doubtless light and inextensible as well - all we really need for this problem is that he's a point!).
The worm begins to crawl, at a speed of 1cm/s along the rope. After 1 second, and after every subsequent second, malevolent fate chooses to take a hand. At the end of each second, the rope is stretched, uniformly throughout its length, to add 1km to its length. Thus, after 1 second, the rope instantaneously becomes 2km long, after 2 seconds 3km and so on.
Hanging on with grim determination, the worm finds himself moving as well. While the distance yet to go has clearly increased, he has been moved forward by the stretching as well.



PROBLEM: <u>HOW FAR CAN THE WORM GET ALONG THE ROPE? ALL THE WAY?</u>
<u>HOW LONG WILL IT TAKE HIM?</u>

*While an interesting exercise in **mathematics**, this is a poor recreational problem; the situation is so **heavily weighted** against the worm, who **seems** to stand no chance at all of **ever making** it to the end, that clearly he MUST make it! Otherwise the shock **to our** intuition that characterises the **best** of problems would be missing. **But it** is very dangerous in mathematics to accept solutions on the grounds that they are merely plausible, and hence doubly dangerous to accept solutions that are implausible. You and the worm both won't get very far if you think about distances. A different perspective will give you a fractionally better chance. The Monument is irrelevant.*

THE WORM THAT DIETH NOT....?

In the worm problem, the rope suddenly expands by 1 km. at the end of every second. A more realistic situation (well slightly more realistic!) would be one in which the rope still stretches by 1 km. per second, but in a continuous and uniform manner. (Imagine that the end of the rope at which the worm starts is fixed, and the other end has a uniform velocity of 1 km. per second away from the fixed point.)

Clearly, in this discrete case, the worm's **arrival at the** end of the rope is something of a **cliff-hanger, in that the series** $1 + 1/2 + 1/3...$ is "only just"**divergent. Could it be** that the **passage** to the **continuous case dooms the worm?**

PROBLEM: DOES THE CHANGE IN THE MODEL CHANGE THE CONCLUSION ?

*If you succeeded **in doing the original problem, and** you **also** know how to set up and solve a **first-order differential equation,** then you should have no difficulty with this **variation.***

Often people don't try to solve problems because they're afraid to fail. The fear comes because they measure themselves by success. What is important is to participate in a problem - solutions or not!

The problems in this book are useless once the solution is revealed. The whole point is the experience leading up to the moment of solution.

A problem isn't solved until you've successfully explained the solution to someone else.

DOTTY

Can you draw four straight lines, without removing pen from paper, so that exactly one line passes through each of the nine crosses in this diagram?

X X X

X X X

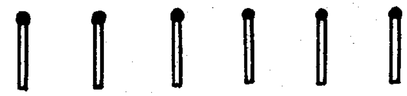
X X X

PROBLEM: WELL, CAN YOU?

This problem has been around a long time, and deservedly so; it is a repeated warning, every time it appears, to would-be mathematicians that it is always dangerous to assume more than you are told. In exactly the same spirit, but discriminating against the novice without a solid foundation in mathematics, is the following....

NEEDLE MATCHES

You are given just six matches, all the same length:



PROBLEM: ARRANGE THE MATCHES TO FORM EXACTLY FOUR EQUAL-SIZED EQUILATERAL TRIANGLES?

Imbued, as you will be by now, with the spirit of not assuming more than you are asked for, please find the following:

PRIME CUTS

PROBLEM I: FIND A FORMULA GENERATING ONLY PRIMES

PROBLEM II: FIND A FORMULA GENERATING ALL THE PRIMES

THE INDUCTION GAME

This game is concerned with infinite sequences of positive integers. I have in mind a certain property which such sequences may or may not have. All I will tell you about it is that there are infinitely many sequences which have the property, and also infinitely many that do not. For example, the property I have in mind might be one of the following:-

1: Each term in the sequence is divisible by 4.

2: Each term in the sequence is larger than the previous term.

Your job is to try to guess the property that I have in mind. To help you guess, you can try to build up a sequence having the property that I am thinking of. Each time you suggest a number that is permitted by the property, I will add it to the end of the sequence, and to help you remember I will write below each term numbers which have been suggested, but which do not satisfy the property <u>at that point</u>. Suppose that the suggestions have gone as follows:

	5	6	3	1	5	11	12	7	?	?
numbers not allowed		4	7 8	4 8 9		3 2	4 3 7 8 10	15 14		

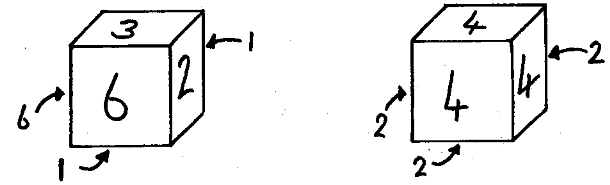
<u>PROBLEM</u>: <u>CAN YOU GUESS WHICH PROPERTY I HAVE IN MIND?</u>

This "Game" genuinely uses inductive rather than deductive reasoning; it also appears as the game of Eleusis, which has had a well-deserved airing in the pages of Scientific American, invented by Robert Abbott. We would encourage you, not merely to solve this single puzzle, but to try the game out for yourselves.

THE GOOD DIE FIRST

A and B are fair dice. That is to say, when you roll them, each of their six faces is as likely to come up as any of the others. However, they differ from normal dice, in that they do not have

distinct numbers on each face. In fact, the numbers on the dice are:



We each take one of these dice and play the following game. We each roll the die we have taken. The player who rolls the higher number receives 10p from the other player. (In the event of a tie, no money changes hands.) If we play this game over and over again, which of these two dice would you rather play with? Try and arrive at an intuitive judgment without making any calculations.

In fact, the calculation is not difficult to make. We need only draw up a table of the 36 possible outcomes (all different and equally likely) and indicate who wins in each case.

		DIE A					
		6	6	3	2	1	1
	4	A	A	B	B	B	B
	4	A	A	B	B	B	B
	4	A	A	B	B	B	B
DIE B	2	A	A	A	=	B	B
	2	A	A	A	=	B	B
	2	A	A	A	=	B	B

We see that A wins 15 times, B wins 18 times and there are 3 draws. Thus, on average, in every 36 throws die B will win 3 more times than die A. So B's long-term advantage is 1 in 12 or 8.333...% Note that this is despite the fact that the total of the numbers on A is higher than the total of numbers on B.

I also have a third die, C. C is also a fair die and it has exactly the same advantage over B that B has over A. That is, on average, in every 36 throws, C will win 18, B will win 15 and there will be 3 ties. So again, C's advantage over B is 8.333...%

PROBLEM: WHAT IS C's ADVANTAGE OVER A?

The point of the problem is that it is not clear how to combine the two sets of data in order to compare C with A. The correct method might be simply to add the two percentage advantages and thus deduce that C has a 16.666...% advantage over A. Alternatively, you might take account of the fact that C wins half the time against B and B wins half the time against A. Combining these, perhaps C wins all the time against A and so has a 100% advantage over A. Or perhaps there is some other method of getting to the answer?

THE STRANGE AUCTION

I have here a £1 note. I intend to auction this note under the following bizarre, but not superficially impossible conditions:

1: Everyone participating must make a bid on each round of the bidding, or else drop out for good.

2: When the bidding stops - when no one else wishes to bid - the last person to bid collects the £1.

3: Unusually, the person collecting the £1 is not asked to meet his bid, but rather the previous two bidders must meet theirs.

For example, if the last three bids are:

Fred: 95p
John: £1
Alan: £1.05

and no-one else wishes to bid, then (unless Fred wishes to up his bid and so on) Alan collects £1, and Fred pays 95p and John pays £1. Alan's £1.05 is a purely nominal amount, which he is not asked to pay - unless that is, the bidding goes further.

PROBLEM: WHY WOULD YOU DO WELL TO AVOID THIS AUCTION?

"The joy of being present at the solution is worth all the frustration that precedes it"

Learning mathematics by listening to a lecture is like learning a game by watching professionals play.

"A problem presents itself - it must be solved. There's a nagging curiosity that won't go away".

MY FAIR LADIES

One of the delights of a Summer School - at least, the newspapers would have us believe so - is the profusion of pulchritude. Shortly, a new style of beauty contest is going to begin. Along this passage will pass TEN beautiful ladies. YOU are going to choose just one of these. The rules governing your choice are extremely simple: as each lady passes you must decide whether to TAKE her or REJECT her. If you take her, then irrevocably she is the one for you - even if the remainder of the ten, previously unseen, all turn out to be the more desirable. Likewise, if you reject her, you cannot subsequently change your mind, even if the unseen beauties all turn out to be less than desirable.

In order to make all of this a little bit more manageable, let us assume that 'desirability' may be represented by a positive real number - a 'score' as it were. As each 'score' is offered to you, you must decide then and there whether to take that score or to hang on in hopes of a higher score. It seems unnecessary to say that you may only take one lady!

PROBLEM: WHAT STRATEGY SHOULD ONE ADOPT IN ORDER TO ACHIEVE THE BEST RESULT ON AVERAGE?

This is rather a nice problem, and not at all an easy one to solve. Firstly, it isn't at all clear what is meant by the "best result". You must decide in your own mind whether the object of this rather artificial (but nonetheless reminiscent) game is to win or to try and maximise your score. The use of strategies, and the notion of 'on average' both suggest that the game will be played, in theory at least, a large number of times. Being a good second each time could be a viable strategy in the second case, but would be useless in the first where you are 'going for the gold' all the time.

The problem is that whereas initially you can be absolutely sure that the best woman (highest score) is still to come but you have no idea what sort of scores are likely, at the end of the game you know exactly what sort of *scores there were but you **are absolutely sure that** the best woman has already gone. Compromise is **indicated.***

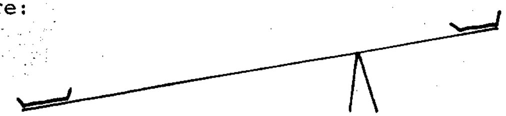
*There are **several entertaining alternatives** to this problem. You might like to investigate **strategies that you would** choose if you:*

*a) **knew that scores lay between** 0 and 1000 only*
*b) **didn't know how many women** were coming*
*c) **didn't know how many women** were coming, but knew that there were at least 10*

*For those **unable to cope with the detailed** mathematics, suggest a strategy in general terms; **for those with a more practical** bent, try a computer simulation.*

SUGAR, SUGAR

They may be canny in Scotland, but they sure are peculiar! I was in Stirling, buying 2lbs of sugar, the other day, when I noticed that the shopkeeper was using a variant on the chemical balance - the variant being that the pivot was most definitely not in the centre:



However, my initial distrust was somewhat allayed when the shopkeeper weighed me out 1lb by balancing the weight in the left with the sugar in the right, and then added to it another 1lb that had been weighed with the sugar in the left and the weight in the right:

PROBLEM: WHO PROFITED: MYSELF, THE SHOPKEEPER OR NEITHER OF US?

This problem hardly takes a moment or two to solve, once you've seen how to do it.

DON'T KEEP ME HANGING ABOUT.....

(The Editor** has been dying to use that title ever since, at the age of 11, he **played the Hangman in the School Play and had only a one-line speech)

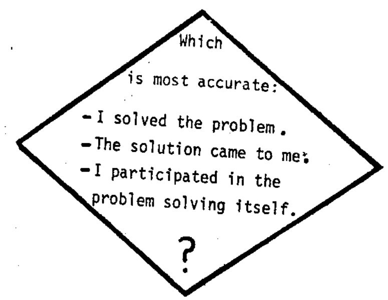
I once found myself in a gaol in an unmentionable country in a state of political unrest. I was put in a cell for two people, and there were two other single cells. Each day, three prisoners, guilty of some terrible crimes, were brought in, one to each cell. They were told that exactly one of them had been pardoned and that the other two would be hung at dawn. The first time this happened, I fell into conversation with my temporary cell mate, and he argued as follows:

"There is an equal probability that any one of us three poor souls has been pardoned, so my chances are one in three." I was convinced. When the guard brought our evening meal I asked the guard which of the three was to be pardoned. The guard replied that he was not allowed to reveal any information. My friend then pointed out that since at least one of the other two would be hung, the guard could safely tell us one name - and he did. My friend then turned to me and said "Aha - there are now just two of us so I have a fifty-fifty chance of being the pardoned one."

On reflection, it seems that he was not given any information, and yet his probability of being pardoned changed

PROBLEM: WHAT DO YOU THINK HIS CHANCES WERE?

***A careful prescription of the sample space** will reveal that a conditional **probability is being calculated here**, and a hidden probability is being varied!*



THE FIFTEEN GAME

This game is played between two players. Nine cards bearing the numbers from one to nine are placed face up on the table:-

1 2 3 4 5 6 7 8 9

The players take it in turns to choose the cards. You win as soon as, among the cards you have chosen there are three whose numbers add up to 15. For example, if the play proceeds as follows:

PLAYER I: 2, 8, 4, 9

PLAYER II: 5, 6, 3

then player I wins, because as soon as he chooses 9 he has three numbers which add up to 15 (= 2+4+9)

Try playing this game a few times, and see if you can work out a good strategy for the players. The real problem with this game is, however:

PROBLEM: HAVE YOU EVER PLAYED IT BEFORE ?

This is a good example of how an unfamiliar situation, when looked at in the right way, can resemble something very similar to everyday experience. A great deal of mathematics is just this: a tool, invented for quite a different purpose, can be seen to have a new and different life.

THE FORTY FAITHLESS WIVES

In a far away country, there was once a small village, and among the married couples in the village, exactly forty wives were unfaithful to their husbands. Gossip in the village was rife, but it was an inflexible rule that a husband was never told about the unfaithfulness of his own wife. As a result, all the men in the village knew, for all the women in the village, other than his own wife, whether or not she was unfaithful.

One day, a great panjandrum came to the village. When he discovered the state of affairs in the village, he gathered together all the inhabitants and made the following announcement:
"I regret to tell you that there is at least one unfaithful wife in this village." After this, he left hurriedly.

Now there was a law in this land, that when a man discovered his wife to be unfaithful then, before the next midnight, he had to kill her. The penalty for breaking this law was so horrible that the law was always obeyed, but since the same penalty was applied to those who killed their wives without knowing them to be unfaithful, no-one ever killed his wife without knowing for sure that she was unfaithful.

PROBLEM: WHAT HAPPENED IN THE VILLAGE AFTER THE PANJANDRUM MADE HIS ANNOUNCEMENT, AND WHEN?

It is necessary to know that, not only was each man in the village an excellent logician, but also that each knew all the others to be excellent logicians too. Thus you may assume that the happenings in the village after the announcement were determined by strict logic and did not occur in any random or haphazard fashion.
You will notice that all the husbands of the unfaithful wives knew all the time that 39 wives in the village were unfaithful. So the panjandrum's announcement that there was at least one unfaithful wife could have come as no surprise. Since it seems that it added nothing to what they already knew (or did it?) it would seem that the announcement would have no consequences, and that the village would continue with its previous happy ways.
If this solution does not satisfy you and you think that the consequences might have been more dramatic, you might find the following two questions helpful in trying to work out the answer:
Is there any symmetry in the problem?
Is there a similar, but simpler, problem which you can solve?
These questions are, of course, among those you should always ask yourself if you are faced with a difficult problem.

More than any other, this was THE problem of Chez Angelique. Apart from the amounts of brain-power expended on it, it is memorable for the man who reduced it to the much simpler case of one married couple, and concluded that the wife had no-one to be unfaithful with!

NUMBERS ON THE BRAIN

Algernon and Bartholomew are seated, facing each other. Each has a piece of card attached to his forehead on which is written a positive integer. They can see each other's number, but not their own. On a blackboard which both can see are written two positive integers.

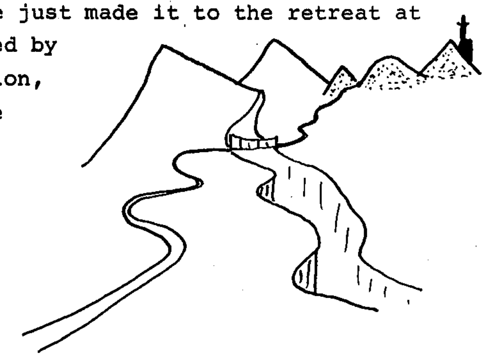
They know that one of these **numbers is** the sum of the numbers on their foreheads, but they do **not** know which it is. They take it in turns to ask each other:
"Do you know the number on your forehead?"

PROBLEM: WILL IT ALWAYS BE THE CASE THAT ONE OR OTHER OF THEM WILL EVENTUALLY BE ABLE TO ANSWER "YES" TO THIS QUESTION?

Notice first that they both know that the sum of their numbers is larger than either of them (positive integers). So as soon as one of them knows that one of the numbers on the blackboard is smaller than (or equal to) one of the numbers on their foreheads, he can deduce that the other number is the sum (and hence deduce his own number).
This problem is similar to the Forty Faithless Wives, since Algernon & Bartholomew are able to make deductions not only from what they can see, but also from the answers of the other.
Once again, it may be helpful to think of a specific, simpler, case, say A=4, B=7, and the numbers on the blackboard are 11 and 15.

THE PERIPATETIC MONK

A monk began one day at sunrise to climb up a mountain to a retreat at the top. He went quickly at first, but as the day wore on he slowed down, finding the sun hot and the path increasingly steep. At noon, he rested by a stream for a while, and then set off again. Taking more and longer rests, he just made it to the retreat at sunset. A week later, refreshed by his period of silent contemplation, he headed joyfully down the same path. This time he stopped occasionally to view with new eyes the wonderful scenery he had overlooked on the way up. At sunset he reached the foot of the mountain.



PROBLEM: MUST IT BE THAT HE WAS AT THE SAME POINT, AT THE SAME TIME ON BOTH THE UPWARD AND THE DOWNWARD JOURNEY?

YES - obviously! This sort of puzzle doesn't get asked unless the answer is 'yes'. Which is a pity, as the solution is a delightful piece of discovery. The next two problems are similar in solution: you may elect to solve the PERIPATETIC MONK first of all, and see if you can easily apply the same techniques, or alternatively

try reading through all three of the problems to try to identify what they have in common. As with many similar problems, a new twist will help.

BALANCING ACT

I have a two-dimensional balance which must be placed absolutely level on the table in order that its mechanism can work properly. Unfortunately, my flat table is perfectly steady, but on an uneven floor.

PROBLEM: CAN I FIND SOMEWHERE TO PUT IT ON THE TABLE, SO THAT IT IS LEVEL?

TRIPOS PROBLEM

Although the edges of my sitting room are perfectly level, the floor has heaved a bit, and is anything but level.

PROBLEM: IS IT POSSIBLE THAT I CAN GET A THREE-LEGGED STOOL TO BE PERFECTLY LEVEL ANYWAY?

It will be a mistake to spend so long attempting to identify the connection between these three problems that you make no serious attempt to solve them. The connection is strongest between the latter two.

ORANGES AND LEMONS

A farmer is taking three crates of fruit to market. One contains oranges, the second contains lemons and the third contains a mixture. However, his small son has switched all the labels round, and has issued a challenge that his father should re-label all three crates correctly, by picking out just ONE piece of fruit, from just ONE crate.

PROBLEM: WHICH CRATE SHOULD THE FARMER CHOOSE FROM?

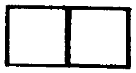
It is rather too easy, in the Oranges and Lemons problem, to forget the most useful piece of information that you have!

HAND OVER HAND

This problem ~~comes~~ in the form of a game involving communication. The rules are as follows:

(i) There must be at least two players,
(ii) No one may use their hands to draw, write or gesticulate.

The question is: "how many different pieces can be formed by glueing up to FOUR little squares together edges to edges?" The rule of combination allows squares to be glued thus:



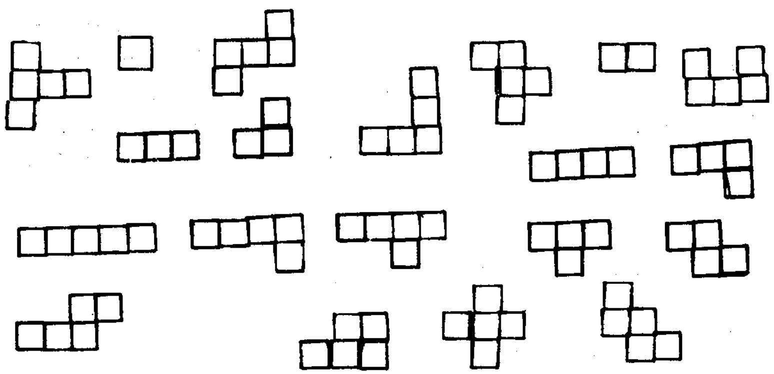
but not thus:

After each person has made an estimate, the players take it in turn to announce and describe (without using hands) a new shape, thus building up a dictionary of names and descriptive adjectives. It is, of course, no use just making up a new name: the notation must be compatible and you must be able to indicate (from memory) why your shape is different from everyone else's.

Oh - what does DIFFERENT mean? The players decide that as they go along.

Communication demands that we choose words which are appropriate to to the listener. In this case, a new word must gain acceptance before it passes into general use. The process is similar to the process of concept formulation. Having tried 4, try 5!

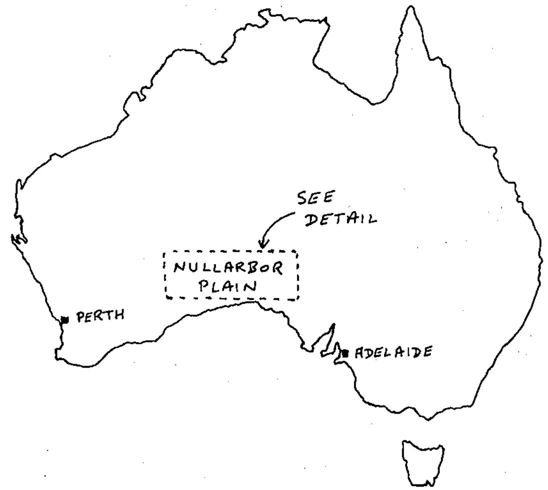
As the 'answers' are not really relevant to an experiment in communication, we give the numbers and shapes for the cases 1 to 5; rotations and reflections are not considered 'different'

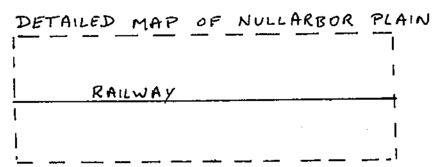


A QUESTION OF LIFE & DEATH

Imagine if you will, finding yourself in the following (unlikely, but plausible) predicament:

Weary of European decadence you decide to emigrate to Australia; your plane lands first at Perth, but you intend to continue your journey to the Eastern states. Unfortunately, between Perth and Adelaide the plane crashes. YOU are the only survivor. It is the middle of a moonless night.
Suddenly, the silence is broken by the whistle of a train!





You must not assume that civilisation is near at hand. Your plane has crashed in the Nullarbor plain. A glance at the map will reveal that the Nullarbor Plain has only one geographical feature - 300 miles of perfectly straight railway line. The train driver was not whistling at a level crossing - he was simply expressing his delight at the prospect of a further 150 miles of perfectly straight railway line across the perfectly flat and perfectly waterless Nullarbor Plain.

You have no water, and trains across the Nullarbor are infrequent. Your only chance of survival is to reach the railway line and to stop the train you have just heard.

Admittedly, your prospects are not bright: the whistle was too brief to enable you to judge the direction in which the train was moving, but you have managed to fix the direction from which the sound came. Of course, the plane crash has left you somewhat disoriented and so you have no idea which direction is North, and the unfamiliar stars of the Southern hemisphere are of no help. It is reasonable to assume that the train is moving faster than you can walk.

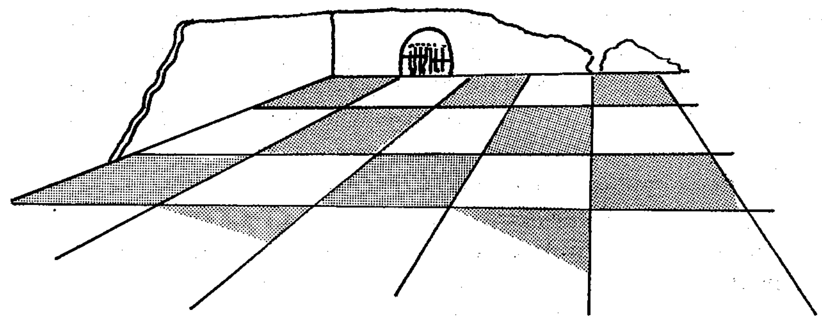
PROBLEM: IN WHICH DIRECTION SHOULD YOU WALK TO GIVE YOURSELF THE BEST CHANCE OF LIVING LONG ENOUGH TO TASTE BEER THAT HAS LURED YOU HALF-WAY ROUND THE WORLD?

To catch the train, you must arrive at some point before the train reaches that point. If you walk directly away from the sound, every point you reach will have the property that you reach it before the train does! Assuming, of course, that you are not actually standing on the line - which highlights one aspect of the problem that confuses most people: you are not searching for a cast-iron solution. At best, you are looking for the strategy that will maximise your chances. On the surface, they seem pretty slim whatever you do. Even if you were on the track, which seems like a best case of some sort or other, there is only a 50-50 chance that the train is not moving directly away from you

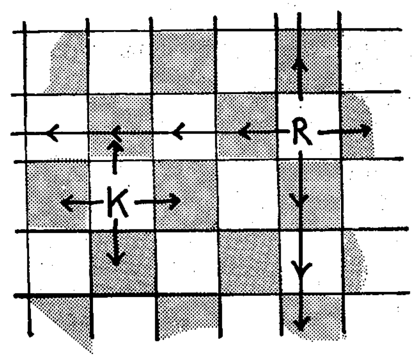
CORRECTION....

The publishers of Chez Angelique wish to apologise: they were not 100% accurate in describing the Nullarbor Plain as featureless - on the other side of the railway track described above in A QUESTION OF LIFE & DEATH, there is a strange, ancient ruin, consisting of two high walls running at right angles to each other from a corner.

The inside has recently been purchased by a rich linoleum tile merchant, who is also a chess addict.



He has arranged for the inside area to be tiled alternately black-and-white, infinitely far in both directions. Of course, the Nullarbor Plain is not in fact infinite, but like most of Australia it seems like it!



As a challenge to the desert wanderer who comes across these ruins, he places a rook in the corner and a king on some (known) square. The moves of these two pieces remain substantially as they are on finite boards - the rook may move any number of squares horizontally (or vertically) in a single move, and the king may move only a single square (horizontally or vertically). Neither piece may move diagonally. By virtue of the rules for taking pieces in chess, the king may not move onto any square which is in the same row or column as the rook, and the rook may not move adjacent to the king. Except diagonally, that is.

PROBLEM: THE KING IS "CAPTURED" WHEN NO MOVE IS POSSIBLE - CAN IT BE DONE?

If it can be done, give an explicit algorithm for the rook. If not give an explicit algorithm for the king to escape.
As a Chez Angelique problem, this was notably unsuccessful: in style, it is a problem that benefits from having a chess-board, and playing around moving a few pieces. Nonetheless, it is an excellent problem on which to cut 'collective' problem-solving teeth. The problem apparently has its origins in computing circles, as does our next one.

FAMILY TREE

Each of us, OU staff included had two parents, four grandparents and, in general, 2^n ancestors, n generations ago. Although this sets an 'upper bound' on the number of ancestors at any given time, clearly the rate of growth of 2^n indicates that not very far back into the past, due to intermarriages, we shared a common set of ancestors. All of which makes something of a nonsense of those people who would claim a pure-bred descent from notables in the past. However, the problem is not about how many ancestors we could have had, but about how FEW!

For the sake of some structure to the problem, let us assume, rather unrealistically, that all marriages have always been within the bounds laid down in the table of 'kindred & affinity' in the book of Common Prayer. Men, for example, may not marry their widows' mothers. (Think about it!)

PROBLEM: WHAT IS THE LEAST NUMBER OF ANCESTORS THAT ONE COULD HAVE HAD ALIVE AT ANY ONE TIME, SAY 200 YEARS AGO?

A strict solution of the problem hinges on the precise definition of alive. A lot may be accomplished by consideration of babies nestling in wombs before birth. In fact, such linguistic tricks don't add much to the problem, and you can set what limits you wish on the definitions, and mark yourself accordingly. Note that you may have no living ancestors, yourself, which is why we set it some centuries back. More usefully, remember that marrying immediately adds to your children's ancestry any living ancestors on the distaff side of the family.....
Not entirely a trivial question, is the following:

PROBLEM: AFTER THE SITUATION OUTLINED IN FAMILY TREE, WHAT HAPPENED?

Nor, surprisingly are these two:

PROBLEM: WHAT IS THE AVERAGE NUMBER OF FINGERS IN YOUR FAMILY (after the events of Family Tree)?

PROBLEM: WHAT IS THE CHANCE THAT THE NEXT PERSON YOU MEET HAS AN ABOVE AVERAGE NUMBER OF ARMS?

ON UNDERWEAR, STRING AND HUMAN BODIES

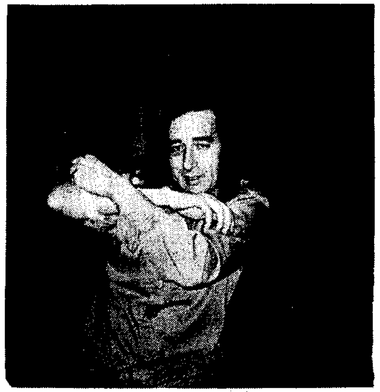
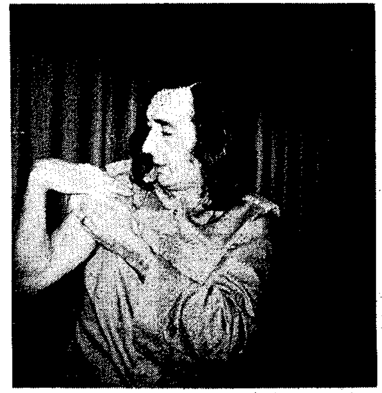
To a topologist, human bodies are rather dull objects, being rather less entertaining than a doughnut or a pretzel. In fact, the topologist sees all human beings as roughly spherical lumps - odd protuberances like head, limbs &c are of no interest and can happily be removed. Of course, not everyone has this unnatural view of humanity. Indeed, many who have made a contribution to society have made reputations by a careful study of just those protuberances that the topologist so carelessly amputates. These latter people are properly described as geometers and among them we find the inventors of the strait-jacket, handcuffs, leg-irons, the thumb-screw, the rack and the guillotine.

The geometry of the human body can provide us with a number of amusing problems. Most of them become more amusing if attempted in front of a small audience prepared to make helpful suggestions. For example, various items of underwear can be removed without removing outer garments. Most women know that this can be done with a bra - it is less well-known that a man can remove his vest without taking off his shirt, although he will probably have to unbutton his collar and cuffs.

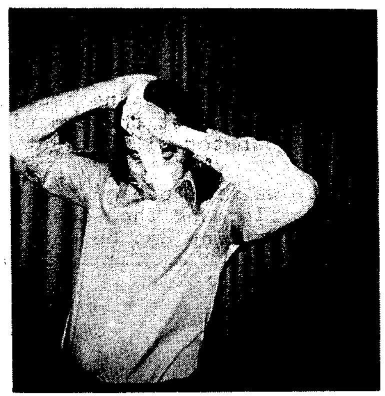
Unable to find a young lady willing to divest herself of a bra, even underneath a dress, while CHEZ ANGELIQUE looked on, but sparing no expense to make this volume complete, we have commissioned a willing volunteer. Before your very eyes on this and the following pages, we show you that IT CAN BE DONE

Problems that involve more than one person can be devised if we allow the use of some string. Take two pieces of string about 4 or 5 feet long and two volunteers. Tie the wrists of one volunteer together to resemble a pair of handcuffs - that is, a loop about each wrist (too small to slide over the hand, but not so tight as to restrict circulation) and a long piece of string joining the two loops. Do the same to the second volunteer, but before tying to the second wrist, pass this string over the first string so that the two victims are linked together. Can the two be separated without untying any knots?

YES: we make use of the fact that there is a small gap between the wrist and the loop of string tied around it. A loop of the other string can be worked through this and over the hand.


CENTRE STRIP!

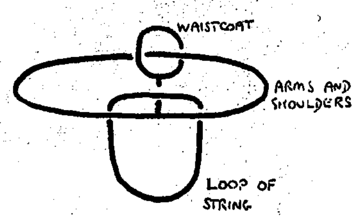



If your circle of friends includes someone who wears a waistcoat, ask him to remove his jacket and then put his arm through a circular loop of string (about 8 feet of string) before putting his thumb into his waistcoat pocket. His thumb must remain firmly in the pocket while the loop of string is removed from his arm. A T-shirt will serve as well as a waistcoat if that's the sort of friend you have. Instruct him to take hold of the body of the shirt after putting his arm through the string.

A variation on this last problem might actually arouse the interest of a topologist. Try putting both arms through the loop of string (from the same side) before attach ing the hands to the waistcoat or T-shirt. Putting both hands through the loop of string and then firmly into both trouser pockets provides a less interesting topological situation, but the geometry is getting better!

For the first version of the problem, take a loop of string through the nearest armhole, over the head, through the other armhole and over the hand. Now pull the string out from the bottom of the waistcoat and down to the floor.

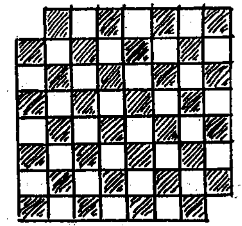
Two arms through the loop? The loop cannot be removed! If we remove parts of the body and waistcoat, we surely don't make the problem more difficult, but we arrive at the following situation where only the arms and shoulders and a strip of waistcoat are left. It is still not immediately obvious that the loop cannot be removed, but the topologists assure us that this is the case.



Two arms through the loop - hands in trouser pockets? Removing the loop without removing the hands is straightforward in theory, but we know of no authenticated case of the trick actually being performed in public.

WE SHALL NOT BE MOVED!

Given a chess-board, and 32 dominoes, of the dimensions shown, it is a relatively easy matter to completely cover the board with dominoes. Rather more challenging is to investigate what happens if we remove the two squares shown from the board. If the board CAN be covered, 31 dominoes will suffice, of course.



PROBLEM: CAN WE STILL COVER THE CHESS-BOARD, EXACTLY WITH 31 DOMINOES?

This is a nice and relatively well-known puzzle. It is a mistake to dismiss it as being colourless and without interest. It may be generalised into the following, equally nice, problem:

AND AGAIN!

Suppose that the two squares removed from the chess-board were not at opposite corners, but rather were in some general position on the chess-board:

PROBLEM: UNDER WHAT CIRCUMSTANCES CAN WE COVER THE BOARD WITH OUR 31 DOMINOES?

This breaks down into two separate cases. In one, the covering is possible, in the other it isn't. Your solution to WE SHALL NOT BE MOVED! will help in the one case, but a constructive proof will be required in the other.

COLOURFUL CUBES

You have a cube and six pots of paint: Red, Orange, Yellow, Green, Blue and Indigo (What, no Violet? We're outa-violet!). With this near-rainbow of colours, you wish to paint your cube with a different colour on each face.

PROBLEM: IN HOW MANY DIFFERENT WAYS CAN YOU DO THIS?

This problem depends to an ***extent on what*** *"different" means. This is a recurring idea in mathematics.* ***The simplest*** *solution, which also gives us an upper bound for all* ***other solutions,*** *is to say that the first face can be any of the 6 colours,* ***but the next*** *can only be one of 5 colours, and so on.* ***In this way we get*** $5! = 120$ *'different' cubes. Allowing more natural interpretations of* ***'different'***, *such as that which says that two ways are different if one cannot be rotated into the other, gives us a more challenging and more interesting problem.*

DRILL AND PRACTICE

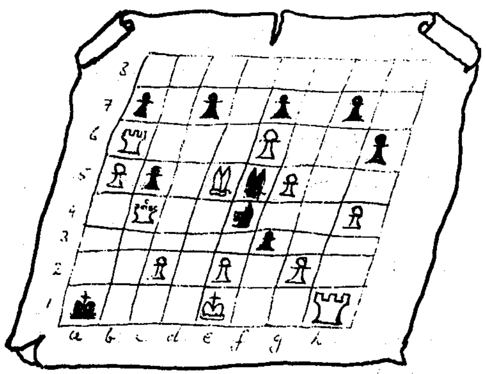
I have a wooden sphere, through which I drill a hole. The hole is of length 2L.

<u>PROBLEM</u>: <u>WHAT IS THE VOLUME OF THE REMAINING PORTION OF THE ORIGINAL SPHERE?</u>

The most glaring omission (for such it <u>seems</u>) in the above problem is that the dimensions of the original sphere have not been given. This, by itself, could be suggestive; the problem changes very much if we assume that there is a solution, because new methods of solution open up.

THE ONE-MOVE MATE

It was breakfast time at 221b Baker Street....
"A gentleman by the name of Collings has just sent me this, Watson. What do you make of it?"
Holmes passed a flimsy sheet of paper across the breakfast table to me. On it was sketched a position in a game of chess.
"You may care to know that the position is one that might arise in play and that in addition, White has moved his QRP twice during the last ten moves....."



There elapsed a period of some minutes while I studied the scrap of paper.
"There are no mating moves, Holmes!" I was forced to cry.
"Come now, Watson! Collings tells us that white moves and mates in one!" I had expected better of you than this. Perhaps you

have not considered:

P x P e.p. <u>mate</u> (a5 to b6)

"Ah yes, Holmes - an <u>en passant</u> capture. I had seen that possibility, but rejected it - we have no way of knowing that Black's last move was:

... P-QN4"

"Except that White <u>does</u> mate in a single move, and once we have eliminated the impossible, all that remains, however improbable, must be .... but I expect that you are quite capable of finishing <u>that</u> epigram, Watson!"
"Simple then, Holmes! There is no other possible move, therefore White takes <u>en passant</u>, and wins..."
"Unless of course, Watson" said Holmes with a touch of a smile flickering about his face, "White castled on the King-side!"

"There are, Watson, two possible mating moves, both requiring some knowledge of how the play has proceeded up to the present position. There are a number of potentially useful observations that we can make immediately"
"I fail to see anything useful, Holmes"
"I commend your attention to the strange affair of the bishop on square f1, Watson"
"But there is no bishop on that square!"
"Precisely, Watson!"

<u>PROBLEM</u>: <u>WHITE MOVES AND MATES IN ONE. HOW?</u>

This problem is delightful! You will deduce from the discussion above that the bishops on d5 and e5 must have been promoted. They clearly could not have left their starting squares. The bishop now on square e5 can be seen to be the promotion of a pawn that captured as it reached the eighth rank. The essence of the problem is to prove that one of the two possible moves must have been impossible, because of the previous game. As pointers to the satisfactory solution, we might draw your attention to the fact that a number of pieces have left the board. Every time a pawn moved sideways, it could only have done so by means of a capture. Hence the pawn moves and the missing pieces are connected.

Perhaps the nicest thing about this problem is that every piece on the board, as well as those off it, plays an important part in the solution of the problem. Good Luck!

THE NON-WORLD OF THE NON-GAME

The idea of a "game" in the classic mathematical sense - an idealised, stylised form of conflict, where outcomes and strategies may be measured, at least in a theoretical sense - is well understood. To some people, this is the single most exciting advance in mathematics within our lifetimes. It is not our wish to quarrel with this idea here.

However, as a means of provoking the reader, and providing a little light relief from the mainstream of problem-solving, let me introduce you to the concept of the NON-GAME.- The Non-Game defies analysis.

NON-GAME NUMBER 1: FINCHLEY CENTRAL

Two players take it in turns to name stations on the London Underground. The first to say "Finchley Central" wins.

Now, just in case you are not sufficiently convinced that there are any problems at all in this game, let me ask you a question: when in the best time to say "Finchley Central"? Clearly, just before your opponent does! The sensitive reader, and who is not, will dismiss as uncultured any strategy that suggests saying FC right away - do as you would be done by, after all!
Russia and America, it could be argued, are involved in a massive game of Finchley Central - spending millions of dollars/roubles of their taxpayers' money on sophisticated systems to ensure that they can say "Finchley Central", or "Armageddon" or whatever, just that fraction of a second before the other.

Non-game the second is closer to our experience of reality:

NON-GAME NUMBER 2: TWEEDLEDUM & TWEEDLEDEE

Dum and Dee are approached one day by the Red King, who offers them £1000 on the sole condition that they agree among themselves on how to divide it. Dum speaks first and argues for 50-50, Dee is slower and more devious, arguing for a 90-10 split in his own favour: "if you don't agree to that, Dum, you get nothing - the choice for you is nothing to do with £1000: it's £100 or nothing!"
Dum is slower still, but still a mathematician: he argues in the same words as Dee - How would YOU advise the pair?

The real answer to the problem is "with caution". To fix it closer in reality, let us suppose that you had developed some device that would benefit (say) the GPO. Suppose it saved them £1 000 000. How much is a fair price for them to pay you? If you cannot sell it elsewhere, as might well be the case with a monopolistic GPO, any price that covers your development costs is profit for you. Likewise, anything up to about £900 000 is a worthwhile profit for them. The only merit in 50-50 is a misguided appeal to symmetry.

A similar, and equally baffling situation arises when Wynken, Blynken and Nod get to dividing - a situation matched only by Dancer, Prancer, Donner and Blitzen....

Less rooted in reality, but in the same vein is:

NON-GAME NUMBER 3: PENNY POT

Players take it in turns to, either, add a penny to the pot, or take the contents of the pot. If you take, you start next.

Work it out for yourselves - a less serious Non-Game that led to a fascinating experiment is:

NON-GAME NUMBER 4: MISS TAKE

A number of 'judges' view a parade of scantily-clad 'beauty queens'. This is just light relief, and has nothing to do with what follows, when the judges all try to choose the same number as all the other judges. No other criteria are operative.

As a result of musing over this 'non-game' I once set a class an examination with a single question - viz: "Who will come top in this test?"
That factors alter cases was admirably proved; Jones, let us call him, had always come top in every other test, so to a man the class wrote "Jones" - except, that is, for Jones, who in a fit of modesty, wrote "Smith"

Lastly, with no explanations:

NON-GAME NUMBER 5: FRUIT MACHINE

Players pay good money to watch some coloured wheels spin around. Nothing else ever happens, except to someone else.

NON-GAMES were described very readably by Anatole Beck in an article in MANIFOLD-4, published at the University of Warwick.

WHAT NEXT ?

110, 20, 12, 11, ? 6, 6, 6, 6, ...

PROBLEM: NOT SO MUCH 'WHATS NEXT' AS WHAT'S MISSING?

SATISFACTION!

Summer 1975 in Stirling was really incredible: those with previous experience of the delightful campus at the gateway to the Highlands will know how unexpected the weeks of unbroken good weather were. Naturally, everyone was even thirstier than usual. Passing between lecture and review session one morning, I chanced upon three students on the grass. They had a large jug of beer, and a single glass. Being totally uncharitable, and very very thirsty, they were trying to divide the jug of beer so that each one is happy that he has a fair share.

PROBLEM: HOW CAN THEY ACHIEVE THIS DESIRABLE END?

Once again, the problem is capable of being misunderstood; the drinkers are not attempting to divide the beer equally, but to divide the beer into portions that each are prepared to accept as thirds. Unfortunately, there's no way to mark the jug, or the glass!

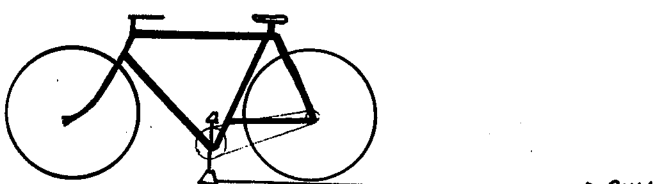
EUREKA !

A small boy is to be found sailing his toy boat in the bath. The toy captain of the boat is made of metal, and falls overboard.

PROBLEM: DOES THE WATER-LEVEL RISE OR FALL?

BICYCLE BALANCING.....

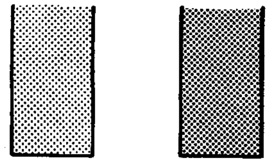
Arrange a bicyle as shown, and pull on the string.



PROBLEM: DOES THE CYCLE MOVE FORWARDS, BACKWARDS, OR WHAT?

AND A WHISKY CHASER, PLEASE

Stirling is the home of whisky - from the top of Dumyat it is possible to see Scotland's largest bonded malt-whisky warehouse. When it isn't raining, that is! The following problem seems rather appropriate:



I have two tumblers, as shown. One is full of whisky, the other has the same amount of water. If I take a spoonful of whisky out of the first tumbler and stir it into the second, and then take a spoonful of the mixture out of the second and return it to the first:

PROBLEM: IS THERE MORE WHISKY IN THE WATER THAN WATER IN THE WHISKY?

Ah! you say - you haven't told me whether the whisky and the water mix perfectly. It could be that you carry back the same spoonful of whisky that you carried over. You could even carry back NO whisky at all, and just take water. Or, probably, you take back some proportion of whisky and water - about 50-50 perhaps, but not surely.
Mmmm, we say......

HOW TO TAKE A PRETTY GIRL TO LUNCH

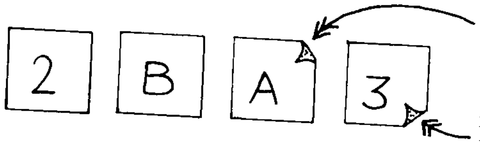
"If I ask you three questions, will you answer them truthfully?"
"Yes, of course!"
"Good - that was the first question! Now, if my next question is 'will you have dinner with me tomorrow night?', will you answer that question the same way you answer this one?"

PROBLEM: WHO'S GOT PROBLEMS???????

Solutions

VOWEL

There is no solution in the strictest sense. It depends on your interpretation of logic in natural language: if, more naturally in this context, you use the "mathematical" interpretation, only two cards need to be turned over.



If this is NOT an even number, the assertion is disproved.

If this is a vowel, the assertion is also disproved.

It is common, and by no means wrong, to interpret the assertion as 'if-and-only-if', in which case all FOUR cards must be examined. This is a succinct reminder that we choose to use common words and phrases with technical meanings in mathematics only at our peril. FUNCTION is anything but functional. INTEGRATION has nothing to do with segregation. CLEARLY means 'I think I could do it if pressed'. TRIVIAL means 'I could do it now, after much thought!'

SQUARES AND CIRCLES

ALL of the cards are still possible solutions! By our definition of agreement, the card could well be a black square, a black circle or a white square. It cannot be a white circle (different colour and different shape), but the white circle can still agree with our fifth card.
All of this leaves us wondering if our question has told us anything at all? In fact, the full information carrying content of this reply can only be used after another question.

POLITICIAN'S PROMISES

Rewrite the statements in their logically equivalent forms: "Four of these statements are true", "Three of these statements are true" etc. The statements are clearly mutually exclusive. It is easier to see that statement 4, which now reads "one of these statements is true" is the true one, than it is to explain why one of them must be true. Basically, this comes about by examining statement 5.

HOW TO GET EVERYTHING WRONG

For n envelopes, the probability P(n) of getting all the invitation in the wrong envelopes is:

$$P(n) = 1/2! - 1/3! + \ldots\ldots \ \ldots + (-1)^n/n!$$

This rapidly approaches e^{-1}, as can be seen from the fact that the n-th order Taylor expansion for e^{-x} is:

$$e^{-x} \cong 1 - x + x^2/2! \ldots\ldots \ \ldots + (-1)^n . x^n/n!$$

and the value P(n) is obtained when we substitute the value x=1 in this expression. In fact, for n = 10, P(n) is already equal to e^{-1} to six decimal places; so the probabilities for n = 10 and n = 1 000 000 are very nearly equal!

To see how to get this result, take the advice given in the hint, and do the calculations for low values of n:-
For n = 2, there are only two ways of putting the invitations into the envelopes - the right way and the wrong way!

For n = 3, there are 3! = 6 ways of putting the invitations into the envelopes, only two of which are all wrong. (There is one way of getting them all right, three ways of getting just one of them right, and it is impossible to get just one of them wrong)

$$P(3) = 2/6 = 1/2! - 1/3!$$

For n = 4, there are 4! = 24 ways of inserting the invitations. Counting how many of these give 'all wrong' is quite complicated, unless you are systematic. The following is a simple and effective system:

Consider the number of ways of getting just one right. If it is the first one, then all the other three must be wrong. We have already seen that there are exactly two ways of getting three items wrong. Thus, there are two ways of getting just the first right, and by symmetry, eight ways of getting just one right.
To discover how many ways there are of getting exactly two right, note that there are $\binom{4}{2} = 6$ different pairs. For each pair that is right, there is exactly one way of getting the other two wrong. Thus there are exactly 6 ways of getting exactly two right.

It is impossible to get exactly three right. There is one way of getting exactly four right. Therefore, there are:

$$24 - 8 - 6 - 1 = 9$$

ways of getting them all wrong.

$$P(4) = 9/24 = 3/8 = 0.3750$$

For n = 5, the procedure is the same.

There are 5! = 120 ways altogether; of these,

$5 \times 9 = 45$ get exactly one right.

$\binom{5}{2} \times 2 = 20$ get exactly two right.

$\binom{5}{3} \times 1 = 10$ get exactly three right.

0 get exactly four right.

1 gets exactly five right.

Therefore, there are 44 ways of getting them all wrong;

$$P(5) = 44/120 = 11/30 = 0.3667$$

It is now not difficult to see how to get a recurrence formula for $P(n)$; Let $N(n)$ be the number of ways of getting all wrong in the n-envelope case. That is, $N(n) = n!P(n)$. Generalizing from the above cases:

$$N(n) = n! - n.N(n-1) - \binom{n}{2}.N(n-2) \ \dots \quad \dots - \binom{n}{n-2}.N(2) - 1$$

If we agree to set $N(1) = 0$, $N(0) = 1$, this can be written as:

$$N(n) = n! - \sum_{j=1}^{n} \binom{n}{j} N(N-j)$$

Using this formula, we get $N(6) = 265$,
$P(6) = 265/720 = 53/144 = 0.3681$

The sequence $\{P(2), P(3), \dots \quad \dots P(n) \dots\}$ is converging remarkably quickly, and has in fact already converged in the first two decimal places. Since the value is just over 1/3, an intelligent guess would be that:

$$\lim_{n \text{ large}} P(n) = e^{-1}$$

In fact, $e^{-1} = 0.3679$ to four decimal places, so already for n=6 we appear to be within 0.0002 of the limit!

Since $e^{-1} = \quad (-1)^n/n!$, it is also an intelligent guess that

$$P(n) = 1/2! - 1/3! + \dots \quad \dots + (-1)^n/n!$$

This guess is actually correct, but unfortunately, I'm damned if I can see any way of proving it using the above recurrence formula! That's why I gave the Inclusion-Exclusion Principle in a rather cumbersome hint.

Consider the set consisting of all possible ways of putting n invitations into n envelopes. Let us label the envelopes from 1 up to n. Let S_1 be the subset consisting of all the ways of putting the invitations into the envelopes in which envelope number 1 contains the correct invitation (but without any presuppositions about the correctness or otherwise of the other insertions). Similarly, define subsets $S_2, S_3, \dots \quad \dots S_n$.

Then the set $S_1 \cup S_2 \cup \dots \quad \dots \cup S_n$ is the set of all possible ways in which at least one envelope contains the wrong invitation. Thus:

$$N(n) = n! - N(S_1 \cup S_2 \ \dots \quad \dots \cup S_n)$$

By the Inclusion-Exclusion Principle, this is:

$$n! - \sum_{i=1}^{n} N(S_i) + \sum_{\text{pairs}} N(S_i \cap S_j)$$

etc....

Now it is easy to see what $N(S_i)$ should be; we make no presuppositions about the other n-1 envelopes, which can therefore be filled in (n-1)! ways. Similarly, $N(S_i \cap S_j) = (n-2)!$, as we are demanding that two envelopes are filled correctly and are making no presuppositions about the others. But there are $\binom{n}{2}$ distinct pairs to be counted; $\binom{n}{3}$ distinct triples; and so on.

Thus the formula becomes:

$$N(n) = n! - \binom{n}{1}(n-1)! + \binom{n}{2}(n-2)! - \dots \quad \dots + (-1)^n$$

$$= n! - n!/1! + n!/2! - n!/3! \ \dots \quad \dots + (-1)^n n!/n!$$

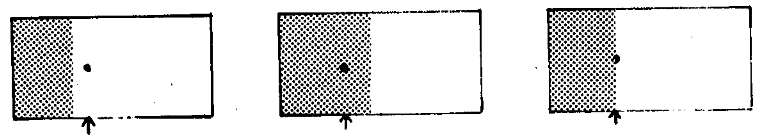
Thus $P(n) = N(n)/n!$

$$= 1 - 1 + 1/2! - 1/3! \ \dots \quad \dots + (-1)^n/n!$$

as was required!

DOWNING ONE'S BEER

Although nothing in the following solution could not be rigorously presented, it seems, from experience, rather easier to explain the solution concisely by executing the following rather drastic steps: freeze the beer, assuming rather unrealistically that there is no change in volume, density etc, and turn the can on its side, balancing it on a knife-edge. There are three essentially different situations:



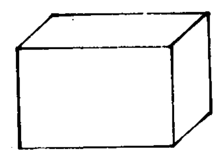
(a) the surface of the beer is to the left of the CG; (b) it is to the right of the CG; (c) it is exactly at the CG.

Take case (a) first. If the air-space to the left of the CG was replaced by beer (presumed denser) then the can would topple to the left, because the CG would have been moved to the left. As this corresponds, in the vertical model, to lowering the CG, clearly the CG cannot be at its lowest point while the surface of the beer is below (to the left of) the CG.

In case (b), if we replace the beer to the right of (above) the CG by air, the can again topples to the left. Once again we conclude that the CG cannot be at its lowest point while the surface of the beer is above the CG.

We are thus forced to conclude that the centre of gravity is at its lowest when it coincides with the surface of the beer

CHEESE SLICES

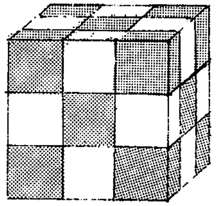


Look at the innermost cube: the one that cannot be seen when the cheese cube is in its uncut state. This cube has six faces. Clearly, no two of these sides could have been produced by the same knife-cut.

Hence, six is the smallest number of cuts necessary.

THE HUNGRY MOUSE

NO. Imagine that the sliced cube is coloured (blue and white, say) so that 'sub-cubes' which meet across a face have different colours. From the outside, the cube would look like this:



The conditions of the problem imply that every time the mouse eats his way across the boundary between cubes, he changes the colour of what he eats from blue to white or vice versa. If there are an odd number of cubes, and he is to finish at the central (blue) cube, then he must also begin at a blue cube, and eat through, necessarily, 14 blue cubes and 13 white ones. Unfortunately, there are 15 blue cubes and 12 white ones - hence no chance for the mouse!

GIVE ME A NUMBER

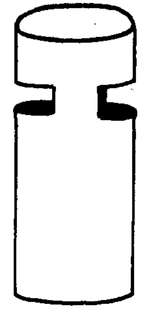
Simply the puzzle works because 7x11x13 = 1001. The apparently innocuous step of 'repeating the digits to make a 6-digit number' simply multiplies the original choice by 1001, and hence makes all the subsequent manipulation possible.
But a simple knowledge of why it works is not the only "solution" to the problem. It is as well to know why it works as well as it does. Firstly, 7,11 and 13 are consecutive primes. If conducted as a group exercise, the skilful manipulator can usually lead his audience into suggesting that last 13 (at least he can ignore any other calls from the audience of possible divisors). Even 7 itself is a 'special' number to many people. Division by 7, 11 and 13 is also just about possible with the equipment that people generally have to hand in mathematical night-clubs (i.e: pencil and paper at most). The 'innocuous' step does, in fact, seem quite harmless. Imagine if you will the following problem, the next one up in the family that this belongs to. It emphasises quite clearly what makes this so attractive a 'quickie'.

"think of a 4-digit number. repeat the digits to make
an 8-digit number. Divide by 73 (and then 137) - what?
no remainder?"

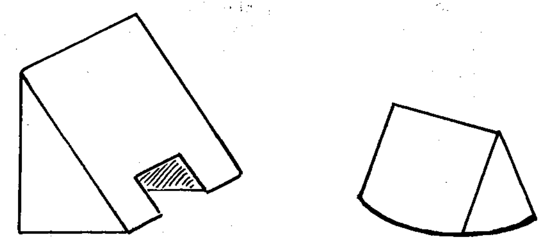
There is so much visible sleight-of-hand afoot (sic) that no-one would be in the least surprised, even if they failed to understand how it all worked.

NEW VIEW

The fuller versions of the objects, fleshed-out to three dimensions are on the top of the next page.



Stirling-type standing ash-trays

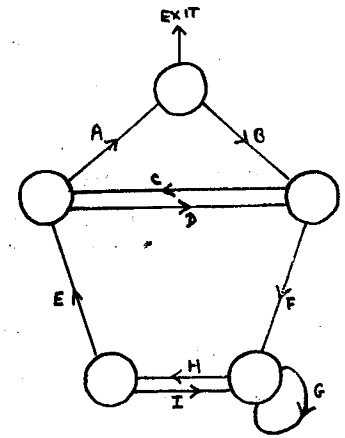


These two are classic design problems.

CAN YOU INVENT ANY MORE? IT'S VERY HARD!

THE AMAZING MANSION MYSTERY

I will show you how to construct a universal escape instruction for the mansion whose plan is shown, but in such a way that it is clear that the same method would work whatever the plan of the mansion.



There are only a finite number of passages in which John can be to send his SOS. For the mansion shown, I have labelled these A,B,.....,H,I. Since there is at least one route to the exit from each room, for each starting position there is at least one sequence of instructions which would take John to the exit. For the mansion shown these are as follows:

PASSAGE	A	B	C	D	E	F	G	H	I
SEQUENCE	L	RRL	RL	LRL	LL	RRLL	RRLL	RLL	LRLL

Here, 'L' stands for left, and 'R' for right. We now consider the passages that John might be in, one by one. If he is in passage A, we see from the table above that the instruction L will get him to the exit. So we begin our universal escape instruction with L. Suppose, however, that he starts in passage B. When he follows the instruction L, he will end up in passage F. We see from the table above that the instruction RRLL would then take him to the exit. So we add these instructions to the one we already have. Thus we get the instruction LRRLL... which will enable John to escape from either passage A or passage B.

Continuing in this way, we next consider what would happen if John starts in passage C. Following the instructions we have already given him, he would end up in passage H. So we now add on the instruction RLL which would get him to the exit from this passage. We now have the instruction LRRLLRLL which gets John out from passages A,B or C. Coincidentally, we see that the same set of instructions will also get him out from passages E,F,G or H. So we need only consider what happens if he is in passage I. Following the instructions we have given him so far, John, starting in passage I, would end up in passage H, so by adding RLL to the end, we get the final set of escape instructions:

L R R L L R L L R L L R L L

which enables John to escape, wherever he starts from. (By considering the passages in a different order, it may be possible to get a shorter universal escape instruction.)

Since there is a universal escape instruction for each mansion, any infinite sequence which contains all possible finite sequences of L's and R's will enable John to escape from any mansion at all. This is because, following these instructions he will eventually reach the universal escape instruction for the mansion he happens to be in, and so will eventually reach the exit and escape. That is, if, fortuitously, an earlier combination of L's and R's does not get him out.

Conversely, it is easy to see that if an infinite sequence of L's and R's omits some particular finite sequence of L's and R's, then it will fail to get out of some mansions. This is because it is easy to construct a mansion such that the exit can only be reached by following some particular finite sequence of instructions just before you reach the exit. (For example, in the mansion shown you can only escape if your last move is to take the left-hand exit.)

THE WORM ON THE ROPE

The worm makes it, but only just! Consider the motion of the worm in terms of fractions of the distance to cover. After 1 second, and just before the rope expands, he has covered 1cm of a total distance of 1km (= 100 000 cm). The rope then expands, and the worm crawls a further 1cm. However, his speed, expressed as a fraction of the total distance/second has dropped. Whereas before it was 1/100 000, now it is 1/200 000. The unit of his speed is "fraction/sec".
Clearly (clearly?) the distance travelled, in this fractional measure, after N seconds, is the sum to N terms of the series:

$$1/100\,000 \;+\; 1/200\,000 \;+\; 1/300\,000 \;+\; \ldots$$

More manageably, this is:

$$1/100\,000\left[1 + 1/2 + 1/3 + 1/4 + \ldots\right]$$

It is a classic result that 1 + 1/2 + 1/3 + 1/4 + ... can be made as large as we wish, by taking a sufficient number of terms. When we have taken enough for the sum to exceed 100 000, the worm will have made it! The time it will take him is beyond belief, and certainly beyond the end of the universe!

THE WORM THAT DIETH NOT.....?

At time t, let x(t) be the proportion of the rope which the worm has traversed. The rope is now of length 1+t km.

There are two ways to find out what proportion of the rope the worm has traversed at time t+h.

Method 1:

At time t+h, the worm has covered a further $10^{-5}.h$ km relative to the rope. Meanwhile, the rope has lengthened to 1+t+h km. Therefore, the extra proportion of the rope which the worm has traversed is somewhere between

$$\frac{10^{-5}.h}{1+t} \quad \text{and} \quad \frac{10^{-5}.h}{1+t+h}$$

In other words:

$$\frac{10^{-5}.h}{1+t+h} \leqslant x(t+h) - x(t) \leqslant \frac{10^{-5}.h}{1+t} \qquad (1)$$

Method 2:

At time t, the worm is (1+t)x(t) km from the starting point. At time t+h, the worm is further from the starting point, partly because of its own crawling and partly because of the motion imparted to it by the rope. Its own crawling causes it to cover a distance of $10^{-5}.h$ km. At any (variable) time t_1, the velocity imparted to it by the rope is equal to the proportion of the rope it has traversed at t_1 multiplied by 1km/sec. Therefore the extra distance from the starting point which the motion of the rope imparts to it, between t and t+h, is somewhere between h.x(t) and h.x(t+h). Thus, at time t+h, the distance d of the worm from its starting point is given by.

$$(1+t).x(t)+10^{-5}.h + h.x(t) \leqslant d \leqslant (1+t).x(t)+10^{-5}.h + h.x(t+h) \qquad (2)$$

Thus, the proportion of the rope which has been covered at t+h is d/1+t+h, and this is equal to x(t+h). Thus, expression (2) becomes:

$$\frac{1+t}{1+t+h}.x(t) + \frac{10^{-5}.h}{1+t+h} + \frac{h.x(t)}{1+t+h} \leqslant x(t+h) \leqslant \frac{1+t}{1+t+h}.x(t) + \frac{10^{-5}.h}{1+t+h} + \frac{h.x(t+h)}{1+t+h}$$

Subtracting x(t) from this expression, and simplifying:

$$\frac{10^{-5}.h}{1+t+h} \leqslant x(t+h) - x(t) \leqslant \frac{10^{-5}.h}{1+t+h} + \frac{h}{1+t+h}.(x(t+h) - x(t))$$

which reduces to the same as expression (1).

Thus, both methods give the same expression for x(t+h)-x(t), and in the limit as $h \to 0$ this gives the differential equation:

$$x'(t) = \frac{10^{-5}}{1+t}$$

This is a first-order differential equation, whose general solution is

$$x(t) = \frac{10^{-5}}{1+t}.dt = 10^{-5}.\ln(1+t) + A$$

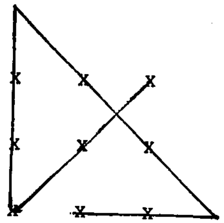
At t=0, the worm has covered a zero proportion of the rope, and so $x(0) = 0$. Thus A=0.
Therefore, when $10^{-5}.\ln(1+t) = 1$, the worm reaches the end of the rope.
This happens when:

$$1+t = \exp(10^5)$$

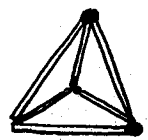
$$t = \exp(10^5) - 1$$

An awfully looooong time!

DOTTY



NEEDLE MATCHES



PRIME CUTS

There is little of prime-number theory here: the sequence defined by

$$u_n = 2 \quad \text{(i.e: 2, 2, 2, 2, 2, 2, ...)}$$

contains only prime numbers; the sequence defined by

$$u_n = n \quad \text{(i.e: 1, 2, 3, 4, 5, 6, ...)}$$

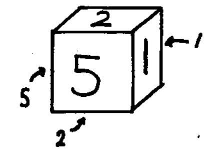
generates <u>all</u> the prime numbers (and a lot else besides!)

THE INDUCTION GAME

In this example, numbers divisible by 3 have to be followed by smaller numbers, numbers not divisible by three have to be followed by larger numbers.

THE GOOD DIE FIRST

The surprising thing is that in fact it is die A which is better than die C! The numbers on C are:



If you repeat the calculations that we did before, you get to the following results, as shown in the tables below.

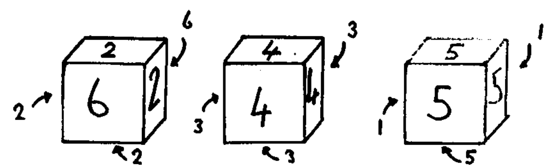
		DIE C					
		5	5	5	2	1	1
DIE B	4	C	C	C	B	B	B
	4	C	C	C	B	B	B
	4	C	C	C	B	B	B
	2	C	C	C	=	B	B
	2	C	C	C	=	B	B
	2	C	C	C	=	B	B

		DIE C					
		5	5	5	2	1	1
DIE A	6	A	A	A	A	A	A
	6	A	A	A	A	A	A
	3	C	C	C	A	A	A
	2	C	C	C	=	A	A
	1	C	C	C	C	=	=
	1	C	C	C	C	=	=

From these tables you can see that when B plays against C, on average, in every 36 throws C wins 18, B wins 15 and there are 3 ties. So C has an 8.333...% advantage over B. However, when we play A against C, we find that in every 36 throws, on average, A wins 17, C wins 14 and there are 5 ties, and so A has an 8.333...% advantage over C.

Thus with these three dice B is better than A, C is better than B and yet A is better than C! Thus the relation "x is better than y" between dice in the sense explained is not a transitive relation. Thus it is perfectly logical for you, given the choice, to prefer B to A, to prefer C to B and yet nonetheless prefer A to C.

The three dice here do not form the only set of intransitive dice. This set was chosen because each of the dice has exactly the same percentage advantage over the die it is preferable to, as do the other two. An alternative set of intransitive dice is shown below. These do not have this property but they have the advantage that with these dice, ties are not possible and the advantages of one die over another is even greater than with the set given above. I will leave it to you to do the calculations for these dice.



You will notice that it doesn't really matter what the proportion of 3's and 4's on die B really is.

If you make a set of these dice and take them into a pub, you can make a lot of money. Play A against B and let people notice that B wins most of the time. Similarly with B against C. Naturally, if you now challenge them to a match of 25 throws and let them choose between A and C, they will choose to play with C. Happy winnings!

THE STRANGE AUCTION

This auction is an excellent demonstration that it is easier to get into trouble than it is to get out of it. Let us assume that the bidding has been in progress for some while, and that most of the players have already dropped out. It makes life a little easier if we assume a constant difference between bids - say 5p - this doesn't change the problem, but saves a constant reference to "a bid of at least £1.26...." Thus, if the last bid was £1.25, the next (if any) must be £1.30.

There are just three players left - Fred, John and Alan, bidding in that order. The last spoken bid (which must have been Alan's) was £5.45. All other participants have dropped out and can play no further part.

FRED: *"This is all getting a bit much for me. I think I'll drop out now, before the stakes get too high - but will there be two other bids, to free me from the obligation to pay up? I spoke in the last round, and bid £5.35, so if John doesn't bid now, I lose out"*

I'll drop out!

JOHN: *"Well now - there's just two of us left. How do things stand? The last two bids were Alan's (£5.45) and my own (£5.40) and the one before that was Fred's (£5.35). What are my choices? I drop out, or I bid.*
If I drop out, then I certainly don't get the pound note. What do I lose? I have to pay £5.40 because then only Alan is left in and he's the last bidder, so he gets the pound.
If I bid, then I'm still in with a chance."

I bid £5.50

ALAN: *"So - John and myself. What do I do? That last bid of Fred's doesn't matter at all to him now - he's out of it.*
The last three bids were John's (£5.50), my own (£5.45) and John's before that (£5.40).
If I drop out, then I don't get the pound note. John gets the note, because he bid last, but I have to pay up my £5.45.
If I bid, then John might not. Then I'd get the pound note. I'd still have to meet that £5.45 bid, but I'd be a pound better off than if I didn't bid."

I bid £5.55

JOHN: *"If I drop out, then I don't get the pound note. Alan gets the note, because he bid last, but I have to pay up my £5.50. If I bid, then Alan might not. Then I'd get the pound note. I'd still have to meet that £5.50 bid, but I'd be a pound better off than if I don't bid."*

I bid £5.60

ALAN: *"If I drop out, then I don't get the pound note. John gets the note, because he bid last, but I have to pay up my £5.55 If I bid, then John might not. Then I'd get the pound note. I'd still have to meet that £5.55 bid, but I'd be a pound better off than if I didn't bid."*

I bid £5.65

Clearly, while the participants reason as they do, there is no reason why they should ever stop bidding without the intervention of some external agency, such as death, a limit to their funds or some similar such calamity.

It appears at first glance as if there is a flaw in the reasoning, in that the participants do not cover all the cases. The analysis covers what happens if they drop out, and what happens if they bid and their opponent drops out. You may well feel that some sleight-of-hand is lurking in their careless neglect of what happens (as it does) if they bid, but their opponent also bids. The reasoning can be tightened up a little by the following style of reasoning:

JOHN: *"If I drop out then I don't get the pound note. Alan gets the note because he bid last, but I have to pay up my 27 million pounds. Suppose I bid. Then the game goes on and either Alan drops out or I get to this decision point with nothing having changed except the sums involved..."*

A mathematical answer is hard to come by. Once the game has started, with two opposing players making bids (we'll discuss players working together in just a moment) then there's no mathematical reason why it should ever end. £27 000 000 is no nearer to £∞ than £1 is. Once the sums bid have passed 60p (so that at least 50p+55p must be paid) the auctioneer cannot lose.

The best strategies, in order would appear to be:

1: Don't play.

2: If you do play, get out while there are at least two others still bidding. You can't of course know when this is (cf. Fred above).

3: If you're left with only a single opponent, cut your losses right away.

There is, however, a more attractive solution. Get into collusion with your colleagues. Bid 5p,10p and 15p in order and then all drop out. Pay up the 15p (=5p+10p) and split the £1 note between you. Then the only person disappointed is the auctioneer!

MY FAIR LADIES

With 10 women, a large envelope-back and some time and energy - or possibly the computer simulation hinted at earlier, you could perhaps work out that the best idea was to let 4 women go past and select the next woman who rated higher than any of the four you had seen so far. If you got to number ten, you must have taken number 10.

This is clearly the style of solution. We must sample the 'population', to get some idea of what to expect in general, and our decision about any subsequent woman must be based on this sampling. If we are 'going for the gold' then clearly we must select a woman better than any in the sample so far, rather than something such as 'any woman better than the average so far'.

The question remains: how many to sample? It is beyond the scope of this booklet to prove that the nicest solution is to sample 1/e of the expected number (10/e = 3.6...).

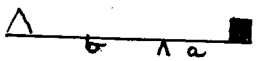
Alternative strategies for those drunk on probabilities could be based on the not unreasonable assumption that the 'scores' are Normally distributed. One possibility could be:

"keep a running computation of the current mean and standard deviation of scores. As soon as one is encountered that is more than 2 standard deviations above the mean (and hence <u>significantly</u> better than the average so far) - take her!"

A not dissimilar problem is that faced by motorists running out of petrol, and searching for a cheap garage. Where do we buy? The cheapest, of course! The problem is slightly different, because of our foreknowledge of the likely expected prices and because we do not know how many garages there will be.

SUGAR, SUGAR

Compute moments - they only take a moment! Look at the first weighing:



From this:

$$1.a = b.w_1$$

$$\text{or} \quad w_1 = a/b$$

Looking at the second weighing:

$$w_2.a = 1.b$$

$$\text{or} \quad w_2 = b/a$$

So, from this:

$$w_1 + w_2 = \frac{a}{b} + \frac{b}{a} = \frac{a^2 + b^2}{ab}$$

$$w_1 + w_2 - 1 = \frac{a^2 + b^2 - 2ab}{ab} = \frac{(a-b)^2}{ab} > 0$$

Hence $w_2 + w_1 > 1$ and it seems that the Scots aren't as canny as they seem!

DON'T KEEP ME HANGING ABOUT

Let my cell mate be A, and the others be B and C and suppose that the guard named B as not to be pardoned. Then the sample space is:

1: A pardoned and B named
2: A pardoned and C named
3: B pardoned and C named
4: C pardoned and B named.

If B or C is to be pardoned, the guard has no choice but to name the other, as he can't name A to A. Thus events 3 and 4 have probability 1/3. However, events 1 and 2 depend on whether the guard names B or C *given* that A is pardoned. Let p be the probability that the guard will name B in this case. Then event 1 has probability p/3 and event 2 has probability (1-p)/3.

Now, we wish to know the probability of A being pardoned, *given* that B is named.

$$\Pr(\text{A pardoned B named}) = \frac{\Pr(\text{A pardoned and B named})}{\Pr(\text{B named})} = \frac{p/3}{p/3 + 1/3} = p/(1+p)$$

When p=1, we get 1/2, corresponding to the guard naming B inevitably, if A is to be pardoned.

When p=1/2, we get 1/3, corresponding to the guard choosing randomly which to name.

THE FIFTEEN GAME

Yes - you have played the Fifteen Game before, and often! It is, in fact, nothing less than noughts and crosses (tic-tac-toe) in disguise. Consider the following magic square:

2	9	4
7	5	3
6	1	8

The numbers in any row, column or diagonal add up to 15. Furthermore, given any three numbers from 1 to 9, which add up to 15, then they make up a row, column or diagonal. Choosing numbers corresponds to putting noughts (for the first player) or crosses (for the second) in the squares. By the remarks above, getting a complete row of noughts or a complete row of crosses corresponds to picking three numbers that add up to 15.

This is a good example of how an unfamiliar situation - the Fifteen Game - can, when looked at in the right way, be seen to resemble a very familiar situation - here noughts and crosses. This is quite a common situation in mathematics where choosing the right model for a problem is often at least half the way to solving it.

THE FORTY FAITHLESS WIVES

On the 40th day after the announcement, all 40 husbands killed their unfaithful wives.
Many people who think that something must happen favour the answer that one unfaithful wife is killed every day for 40 days, but consideration of the first 'question-to-ask-oneself' shows that this answer must be wrong. The situation is exactly the same for each unfaithful wife, so, by symmetry, if any one of them is killed they must all be killed and all on the same day.

To help tackle the problem, it is a good plan to take account of the second question. Let us consider a similar problem with rather fewer faithless wives, say only two. In this case, each of their husbands knows of one faithless wife and so at first is not alarmed by the announcement that there is at least one unfaithful wife in the village. So nothing happens on the first day. But now consider what one of these husbands can deduce on the second day. He can argue as follows: "If my wife is faithful, the husband of the unfaithful wife I know of would not have known that there were any faithless wives before he heard the announcement. So the announcement would have enabled him to deduce that his own wife was unfaithful and he would have killed her yesterday. Since she is still alive, it must be that my own wife is unfaithful to me. Where is my knife?"
Of course, both husbands would argue in the same way and come to the same conclusion at the same time. So both wives would be killed on the second day.

Although it is far from true that a theorem that is true for $n=2$ is true for all values of n, we should have enough confidence now to attempt to prove, by induction, that for all $n \geqslant 2$, in a village as described above, but with n faithless wives, they would all be killed by their husbands on the n-th day after the public announcement.

We have already checked that this is so when $n=2$. Rather than give the general induction step from n to $n+1$ let us, in the spirit of the problem, give the argument from $n=39$ to $n=40$ (it is easily seen that this generalizes).

Thus we assume that we can deduce that in a village with 39 faithless wives they will all be killed by their husbands on the 39th day after the public announcement. Not only can we deduce this, but so too can all the husbands, since they are all excellent logicians.
Now consider the village with 40 faithless wives. Each of their husbands can argue as follows: "If my wife is faithful I live in a village with 39 faithless wives and so on the 39th day after the announcement, all will be killed by their husbands. I shall wait and see if this happens." Thus, they all wait for 39 days, and when none of the wives is killed they can all deduce that they live in a village with 40 faithless wives and hence that their own wives are not faithful. Therefore, on the 40th day, every husband killed his wife.

This puzzle turns up in several different versions. For example, there is the problem of the three people seated around a table with a disc fixed on each forehead. Each can see the other discs, but not his own. They are told that at least one disc is white (and in fact all three are). Can they deduce the colours of their discs?

For a similar, but slightly different problem, try Numbers on the Brain which follows immediately.

NUMBERS ON THE BRAIN

One player or other will eventually be able to deduce his number. Before giving the general argument, we deal with the special case mentioned in the problem. If Algernon, as he must, initially says that he does not know his number, Bartholomew can argue as follows: "Since I can see 4 on A's forehead, and 11 and 15 on the blackboard, my number is either 7 or 11. Suppose my number is 11. Then Algernon, seeing 11 on my forehead and 11 and 15 on the blackboard would know that 15 must be the sum of our two numbers (since the sum must be larger than either of our numbers), and so he would have known that his own number was 4. Since he did not know, my number must be 7."

Bartholomew was able to deduce his number in this case as soon as Algernon said that he did not know his number, because A's number is not larger than the difference (15-11) of the two numbers on the board.
This will always be the case. Suppose, for example, that their numbers are a and b respectively, and that the numbers on the blackboard are x and y, with x greater than y. Our assumption is that A's number is no larger than the difference between x and y - that is, we are assuming that $a \leqslant x-y$. Again suppose that Algernon says that he does not know his number. Then B can argue as follows: "Since I can see a on A's forehead and x and y on the blackboard, my number is either $x-a$ or $y-a$." By our assumption, $y \leqslant x-a$, so B can now say "If my number were $x-a$, then Algernon seeing $x-a$ on my forehead and x and y on the blackboard would know that x must be the sum of our two numbers and so he would have known that his number was $x-(x-a)$, i.e: a. Since he did not know his number, mine must be $y-a$!"

Thus, if A having said "No" gets the same answer from B, he can deduce that his own number is larger than the difference between the two numbers on the blackboard, and B can make the same deduction. Hence after three consecutive "No"s each knows that his number is larger than this difference. Suppose that this difference is k. Then of course, if either number on the blackboard is less than $2k$, each can deduce that the other is the sum of their numbers, and hence deduce his own number. If both numbers on the blackboard are at least $2k$, then each player mentally supposes that the numbers on their foreheads are each reduced by k, and the numbers on the blackboard correspondingly by $2k$. If either player can deduce his own number in this derived game, then by adding k he can deduce his number in the original game. If not, they make a further reduction of k (imaginary) to their own numbers, and $2k$ to the board, and continue in this way. Since their numbers are steadily being reduced, while the difference k remains constant, eventually they are imagining a game in which one or other of their numbers is less than the difference, k, between those on the blackboard. We have seen that in this case, one player or the other can deduce his own number. By adding the appropriate numbers of k's, he can thus deduce his original number.

THE PERIPATETIC MONK

Yes. Let two monks set off, on the same day, one from the top, the other from the bottom, following the exact timing of our monk. Clearly they must meet since they are on the same path and end up in opposite positions at sunset.

BALANCING ACT

Yes. Place it anywhere on the table and rotate it through 360° on the table top. If it starts unbalanced, it ends unbalanced but with the opposite pan high to the one that started higher. Somewhere they must have been level. Compare the Peripatetic Monk.

TRIPOS PROBLEM

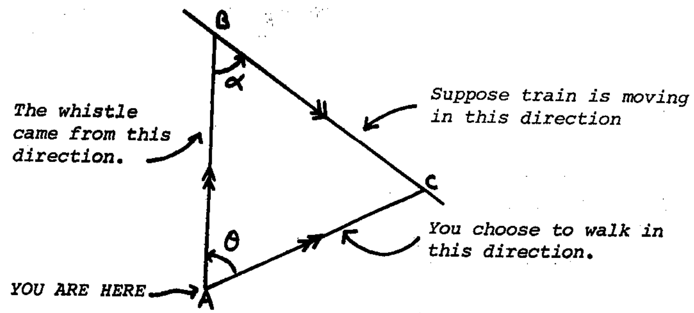
Yes. Same trick as the Balancing Act, but harder to see

ORANGES AND LEMONS

The most important piece of information we have is that all the labels have been switched, and hence no crate is currently labelled correctly. Choose from the crate labelled "MIXTURE". Certainly, it cannot be a mixture, and hence contains just one fruit - exactly which can be determined by the sample. Say it is "LEMONS". Then the crate currently labelled "ORANGES" - which must be either "LEMONS" or "MIXTURE" in reality - cannot be "LEMONS". It must be "MIXTURE" therefore, and the remaining crate, labelled "LEMONS", must be the only other possibility, "ORANGES".

A QUESTION OF LIFE & DEATH

First look at the ground. If you are standing on the line stay where you are and hope that the train is coming towards you. If you are not on the line then, as it is absurd to walk either directly towards or directly away from the sound, you must choose to go either to the right or to the left. Suppose you choose to go to the right (and there is no evidence that will enable you to do better than to choose at random). Then you might as well assume that the train is travelling in some favourable direction, like the one shown in the diagram. (Unless the angle α is acute, the situation is quite hopeless).



It is now up to you to choose a value for the angle θ. The best value for θ is the one which makes the ratio BC/AC as large as possible.

By the Sine Rule (there - you knew it would be useful!) we know that:

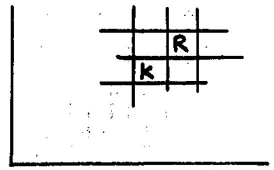
$$BC/AC = \sin\theta/\sin\alpha$$

We can do nothing about the value of $\sin\alpha$. The train has chosen that angle, so we must make $\sin\theta$ as large as we can. Since $\sin\theta$ is a maximum when θ is 90° the solution to the problem is to walk at right angles to the direction from which the whistle came.

Some people are unhappy at walking in a straight line, and propose any one of a variety of spiral-type solutions. It seems self-evident that if you have a strategy that will take you to the train, it is best to get there as fast as possible, but the details of proving that walking in a straight line is best are left to the reader.

CORRECTION....

Once the rook is in the following position: there is no difficulty. The king can only move into the corner, and the rook can follow him, steadily reducing the area that the king has to play with. The rook can reach this position in two moves (provided that he knows where the king is!).



There are other allied problems - what happens if the king and rook make their normal chess moves? Can it be done with a bishop and a king? What if the king can only move diagonally?

FAMILY TREE

The 'minimal' solution is TWO. The two must be women, they must both be pregnant by husbands who have since died and each must be carrying twins. Who carries what is unimportant, provided that the children consist of two boys and two girls. What happens then is that these children must marry, and more restrictively, up till the present day, only cousin marriages can occur. Some medical opinion holds that with such close marriages occurring over a lengthy period of time, polydactylism will occur - that is to say, it is quite likely that descendants of this family will have more than 5 fingers on each hand.

As we hinted, a lot depends on whether you are prepared to accept that a foetus or an embryo is alive before birth. It doesn't change the problem much, either way.

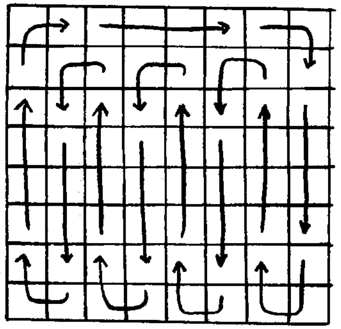
The last problem of this quartet is not on the same theme at all, really. Basically, the argument goes: 'the number of people with more than 2 arms is either zero, or at the least negligible, while the number with one or no arms is relatively high. Hence the average number of arms is less than 2. It is almost certain, therefore, that the next person you meet has an above average number of arms.

WE SHALL NOT BE MOVED!

No! A domino, no matter how it is placed on the board, must cover a white square and a black square. 31 dominoes will thus cover 31 white squares and 31 black squares. But the squares that have been removed were both white squares, so that the mutilated board contains 32 black squares and 30 white squares. No arrangement of 31 dominoes could ever cover this collection of black and white squares.

AND AGAIN!

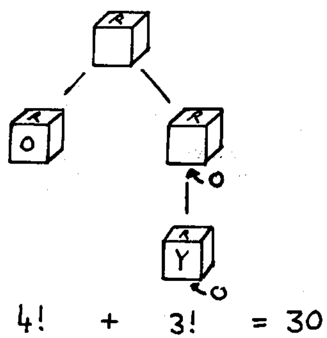
Clearly, from the reasoning in the previous puzzle, if the two squares to be removed are both the same colour, the covering is not possible. To show that it is always possible if the two squares removed are of opposite colours, we arrange the board in a 'chain' like this. White and black squares are seen to alternate along the chain.



When two squares of opposite colour are removed, they are either adjacent in the chain (in which case they could have been covered by a single domino, and any covering of the remaining squares will suffice) or they are separate. As they are of different colours, it will be seen that the effect of removing two squares is to break up the 64-square long black-and-white chain into two even chains: that is, two shorter black and white chains with an even number of squares in each. More precisely, there will be 2n squares in the one chain and 62-2n squares in the other. It is easy to see that it is possible to cover each of the shorter chains. Under these circumstances, the chess-board can always be covered!

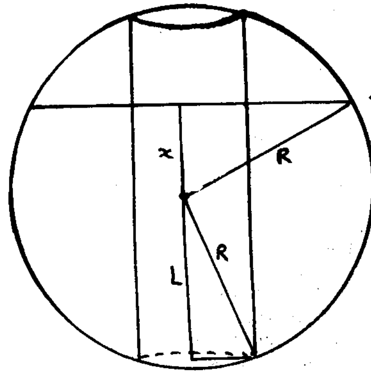
COLOURFUL CUBES

It is always possible to arrange a painted cube with the Red face uppermost, if we accept that rotations, about any axis, do not change the cube into a 'different' cube.
With the cube arranged with the Red face uppermost, the Orange face can be in either of two essentially different positions - on the bottom, opposite to the Red, or on one side. Let us take these cases in turn. If the Orange face is on a side, it can always be turned to face front, still keeping the Red face uppermost. But every different arrangement of the remaining four colours will give us an essentially different cube, so that there are 4! = 24 'different' cubes with Orange on a side.
With Orange on the bottom, we turn our attention to the next colour, Yellow. This must necessarily be on a side, and again we can turn it so that it is to the front, without moving the Red and Orange faces from their positions on the top and bottom. But again, the 3! = 6 possible ways of colouring the remaining 3 faces will give us essentially different cubes.
There are thus 24+6 = 30 'different' coloured cubes. The method of solution, which is just a very careful enumeration of cases, is best illustrated in the accompanying diagram, which says it all!



Compare the enumeration here with that carried out in the problem of HOW TO GET EVERYTHING WRONG...

DRILL AND PRACTICE



The cross-section is an annulus:

whose outer radius is $\sqrt{R^2 - x^2}$ and whose inner radius is the radius of the hole: $\sqrt{R^2 - L^2}$

Thus, the area of the annulus is: (R^2-x^2)

$$\pi(R^2 - x^2) - \pi(R^2 - L^2)$$

which is $\pi(L^2 - x^2)$.

Now this is the same form as $\pi(R^2 - x^2)$, the area of the cross-section of the sphere radius R at length x. So, $\pi(L^2 - x^2)$ is the area of the cross-section of a sphere of radius L at height x from the centre. So for each height x, the annulus area is the same as the corresponding cross-sectional area of a sphere of radius L. It seems reasonable then to deduce that the volume of what remains of the sphere is just the volume of a sphere of radius L (i.e: $4\pi L^3/3$) corresponding to the limiting case of a hole of length 2L through a sphere of radius L. So the volume remaining is constant.
This introduces another method of solution. If this is a well-set problem, and has a solution, then the solution cannot depend on R, for we have not been given R! Let us therefore choose a suitable value for R, to make life easy, such as R=L. QED!
Nonetheless, try the given method with an octahedron and a square hole of length 2L through it. Can you generalise this result?

THE ONE-MOVE MATE

Only one black pawn is missing, and this became the bishop (e5). We shall show that on its promotion route this pawn checked the White king, thereby causing it to move; thus, 'white castles' is no longer a legal move. More strictly, we shall assume that the pawn avoided giving check, and then demonstrate that the board position is impossible on this hypothesis.

The natural and obvious candidate for promotion is Black's QP. To avoid giving check, this must have promoted on c1 (having captured from b2) or a1. In either case, three pawn captures are required. In addition, the bishop originally on f1 must have been captured by a knight, and this accounts for all four captures by black.

The obvious candidate for the White promotion is the QNP, and this must have captured three times to by-pass the black pawns and reach a white promotion square. Thus the tally of White captures is:

3 by QNP +	1 by P(e6) +	2 by P(f5) +	1 by N(say)	= 7
	creating doubled P's	P inversion	capturing B	

But this is impossible as White only made 6 captures in all!

"That completes it!" I cried. "Black's QP must have checked the White king, and castling is illegal. The solution must be P x P e.p. A wonderful example of your analytic skills, Holmes."

Holmes looked displeased rather than flattered...

"When have you known me satisfied with the superficially obvious? Besides, if you had more of the analytical prowess you keep extolling in me, you would not have overlooked an essential part of the data. White's QRP moved twice in the last ten moves."

It is natural to assume that Black's QP promoted, but it is not certain that it did so. It could prima facie, have captured its way to b5, leaving the QNP to promote on a1. If the NP captured away from the b file before the QP captured on to it, White's QNP would have had a clear run to b7 and the earlier deductions would not hold. The captured White pieces consist of the bishop on f1 (taken by a knight), a black square bishop which could not have been taken by the supposed QP at b5, and two knights. Therefore the QP took both knights and the QNP took White's 'black' bishop; but where? It must have been after White's QRP reached a4, but before White's QNP reached b4. Furthermore, P(b5) must have captured from c6 after White's QNP got past. Counting 10 moves back from the board position gives:

	1: P(a5) - a4	
	2: B(d5) - a8=P	
	3: P(a8) - b7	uncapturing a rook; Black P(b5) uncapturing N.
meanwhile, B(e5)	4: P(b7) - b6	
unpromotes on a1	5: N(b5) clears out of way	
to a P which backs	6: Q(b4) clears out of way	
to a3. Black's K	7: P(b6) - b5	
has moved aside	8: P(b5) - b4	
to allow this.	9: P(b4) - b3	Black P(a3) - b4 uncapturing bishop
	10: B(a3) clears out of way	
	11: P(a4) - a3	

Thus QRP can have moved twice in the last 11 moves, but NOT in the last 10. Hence it was Black's QP which promoted, and so the White King has moved.

Holmes sat back in his chair, but was still brooding. "The position is still interesting. If we make the simplest of changes, moving P(c7) to d7, White again has a mate in 1. But now the outcome is different. I wonder whether that fellow Collings is aware of this final twist?"

WHAT'S NEXT?

10, which is "6" in base 6 (after 6 in base 2,3,4 and 5 &c)

SATISFACTION

There are two essentially different approaches. One consists of any one of the three participants pouring from the jug to the glass. As soon as any of the three feels that the glass contains a measure that he would be prepared to accept as a third, he calls out. The first one to call out drinks that third, and the problem is reduced to the two-person case. As the two 'silent' partners have not yet called out, clearly they believe that the first person to speak has drunk less than a third.

An alternative approach, that avoids problems over simultaneous calls, and seems somewhat more workable in practice, is for the first participant to pour out what he believes is a third. If the second drinker disagrees, he is entitled to reduce the measure by pouring back as much as he wishes. If he believes that the first player has poured too little, he will presumably leave the short measure alone - recall that the drinkers are uncharitable. Similarly, the third drinker may alter the amount as he thinks fit, but only by pouring back. The last player to touch the jug drinks the glass and the problem reduces to the two-person case.

If the aim is to divide the drink exactly into thirds (rather than assuming that each drinker acts to maximise his portion, subject to his companions allowing him only one-third) then it is necessary to allow for drinkers to pour into the glass to top-up a deficient 'third'. In that case, the jug and glass must pass a full round without changes.

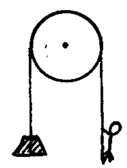
In the two-person case, this reduces to the well-known: "you pour and I'll choose."

EUREKA!

The water-level falls. The simplest explanation of this introduces an excellent technique in that important area of mathematics - convincing yourself that the solution is correct, so that it is worth your while working towards a formal solution!

The technique is called 'taking things to extremes'. The problem depends on the relative weight and volume of the captain. Assume him to be a minute model made out of very dense metal -- consider him sculpted out of one of those incredibly dense stars if you wish. His volume is negligible, but his mass, and hence his weight, is enormous. In the boat, his weight forces the boat into the water, raising the water-level to the point at which we start the problem (remember what Archimedes said about why boats float). As he falls overboard, the effect of his weight is lost, and the water-level falls. The addition of a negligible volume to the water may be neglected, so the water-level stays down.

A regular companion to this problem is the one about the monkey hanging in equi-ibrium in the following situation: The rope is assumed to be light and in-extensible, and the friction at the pulley may be neglected. The problem remains - what happens if the monkey starts to climb up the rope?



This we leave as an exercise for the reader!

BICYCLE BALANCING

Try it for yourself and see! "It falls over" is not an acceptable answer. The hand of God keeps the bicycle upright, throughout.

AND A WHISKY CHASER, PLEASE

The volume in each tumbler is the same at the finish as it was at the start. Irrespective of what has been transferred, what is NOT whisky in the whisky tumbler must be water. And vice-versa. There is exactly as much whisky in the water as water in the whisky.

There are many books on recreational mathematics. It seems invidious to recommend particular ones. However, the reader should find something to his taste in the pages of:

Mathematical Recreations & Essays: W.W. Rouse-Ball (Macmillan)
The Gentle Art of Mathematics: D. Pedoe (Penguin)
Mathematical Puzzles & Diversions from Scientific American: M. Gardner (Penguin)
More Mathematical Puzzles and Diversions from Scientific American M. Gardner (Penguin)
How to Take a Chance: D. Huff (Penguin)

In addition, Scientific American runs a regular monthly column "Mathematical Games" on recreational mathematics.

The journal MANIFOLD, published at the Mathematics Institute, University of Warwick, Coventry CV4 7AL, also carries pieces on recreational mathematics.

'New Scientist' 'The Sunday Times' and other journals carry a weekly recreational problem.

CHEZ ANGELIQUE: THE BUMPER LATE-NIGHT PROBLEM BOOK, is published by Chez Angelique Publications. enquiries and orders should be addressed to John Mason, Mathematics Faculty, the Open University, Walton Hall, Milton Keynes.

JOHN JAWORSKI is a part-time tutor and counsellor with the Open University. He is course editor of the Hertfordshire Computer-Managed Mathematics Project. He has been with the OU since the start, and has worked on courses M100, M251 and TM221.

ALAN SLOMSON is a part-time tutor with the Open University. He is a lecturer in mathematics at the University of Leeds, with a special interest in mathematical logic. He has been with the OU since the start and has worked on courses M100, M201, M202, M331 and M332.

JOHN MASON is a member of the mathematics faculty at the Open University. His particular interests include the nature of learning and collective problem solving. He joined the OU in 1970 and has worked on the course teams for M100, M201 and M202. He is currently involved with M101.

Other puzzles in this book have been contributed by Angela Dean, Fred Holroyd, also from the mathematics faculty at the OU, and by Richard Ahrens, Staff Tutor in the London Region.

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