Editorial Manager(tm) for Educational Studies in Mathematics
Manuscript Draft

Manuscript Number: EDUC913R3

Title: Comparing theoretical perspectives for describing mathematics departments: Complexity and activity

Article Type: Original Research

Keywords: Mathematics departments, Activity theory, Complexity theory, systems

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Abstract: We draw on two studies of mathematics departments in 11-18 comprehensive maintained schools in England to compare and contrast the insights provided by differing theoretical perspectives. In one study activity theory was used to describe common features of the work of three departments. In the other, a mathematics department was viewed and analysed as a complex system. In both cases it was the learning of the departments as systems rather than of individuals that was of interest. The affordances and limitations of the analytical perspectives are discussed. Taken as a whole, this paper opens up the workings of school mathematics departments in a country which has a strong department culture.

Response to Reviewers: The in text (p.4) reference to Williams, Linchevski & Kutscher (2007) has been changed Williams, Linchevski & Kutscher (2008). Some other details of this reference in the list have also been amended.

Thank you to you and the reviewers.

Comparing theoretical perspectives in describing mathematics departments: Complexity and activity

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We draw on two studies of mathematics departments in 11-18 comprehensive maintained schools in England to compare and contrast the insights provided by differing theoretical perspectives. In one study activity theory was used to describe common features of the work of three departments. In the other, a mathematics department was viewed and analysed as a complex system. In both cases it was the learning of the departments as systems rather than of individuals that was of interest. The affordances and limitations of the analytical perspectives are discussed. Taken as a whole, this paper opens up the workings of school mathematics departments in a country which has a strong department culture.

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In this paper, secondary school mathematics departments are seen as identifiable systems, operating with a purpose that distinguishes them from other groups of people within their respective schools. It is customary in English secondary schools for subject teachers to be organised into subject teams, which operate at a middle management level in schools. Although mathematics teachers may have other roles, such as teaching other subjects, or undertaking management responsibilities outside the teaching of mathematics, they belong to the mathematics department with respect to their work of teaching the subject. Departments concerned with teaching different subjects may operate in similar ways for many purposes, such as putting school policies into practice, responding to timetable designs, preparing reports, reporting assessment information and so on, but we might expect them also to be distinguishable through characteristic epistemic cultures (Knorr-Cetina 1999). The concerns of mathematics departments might therefore have some things in common with other groups of people concerned with mathematics just as art departments might have some things in common with other groups of people concerned with art. For example, in comparing the mathematical activity of mathematicians with that in school mathematics classrooms, Burton (2002) noted that one could expect a common appreciation of the aesthetics of mathematics, and shared engagement in seeking connections among mathematical ideas, so we might expect mathematics departments to be distinctive in ways which depend on these appreciations. We also assume that they would be distinctive in ways which relate to current issues in school mathematics teaching in England:

- there is a shortage of mathematics teachers
- there is a high turnover of mathematics teachers
- there is pressure for results because schools are compared using mathematics test results; mathematics has a high political focus
- there are particular inherent cognitive and emotional difficulties in teaching and learning the subject

While in many countries subject departments operate in similar ways to those described here, in others mathematics teachers only come together for particular projects or voluntarily, if at all. In this paper, all aspects of department activity have to be seen as operating within overall school management, rather than purely as collegial or professional development units, so the insights we gained about systemic collaboration sustained over time could be valuable beyond this context. Activity theory and complexity theory offer two different ways of describing and analysing systems. In this paper we briefly describe salient features of each, outline their respective use in two studies of mathematics departments, and compare what each offers as a theoretical perspective through which to analyse school mathematics departments, seen as systems, and looking for manifestations of the epistemic and situational assumptions above. This paper complements studies of how individual teachers learn within departments (e.g. Hodkinson & Hodkinson 2004; 2005) which find that individual learning is often more valued, varied and embedded in collaborative departments than in isolated work situations. However, our intention is to get beyond surface descriptions of collaboration to learn more about how it works in relation to change.

Our decision to compare the affordances of two different theoretical frames arose from a research problem. Two of the authors had used activity theory as an analytical perspective to gain an understanding of the work of three mathematics departments who were engaged in changing their practice over three years. This perspective allowed us to identify sequential and systemic change but we wondered, retrospectively, if a complexity theory perspective in which the maintenance of diverse interpretations and simultaneous change are characteristics of the system might have helped us view these differently. The original research sites were no longer available to us, and we did not have appropriate data from them, so we found a new site to test the conjecture that complexity theory makes more sense of the diversity in mathematics departments during times of change. As we shall show, complexity theory does not presuppose the existence or emergence of particular features and hence to the extent that it is possible data collection needs not to be constrained or subtly directed by assumptions of structural features inherent in perspectives such as activity theory. Rather, it must allow for unpredictability by beginning with very broad ideas and allowing the participants to guide developing focuses.

Analysis of subject departments is relatively new, and we hoped that juxtaposing two analytical methods, albeit in two different situations, would open questions for future research as well as shedding light on aspects which might escape methods arising from only one perspective. Even and Schwarz (2003) use two interpretations of the same lesson, from cognitive science and activity theory, to demonstrate that different theories ask different questions and may provide different 'reasons' for similar phenomena. Because the 'answers' provided by the perspectives are very different they suggest that theory and research can become locked into a 'vicious circle' in which findings necessarily support the theories that led to their construction. In this study we are not seeking answers however; we deliberately set out to use two different methods of analysing the behaviour of people organised into purposeful groups to see what each method affords for understanding subject departments. Due to the limitations of space we focus on post-analysis features of department work, rather than starting with empirical data.

1. Activity

Activity theory, used here as a way to model how systems change, focuses on structured features, identifies the ways in which they interrelate, and sees how tensions provoke change (Bakhurst, 2009). Activity consists of a group of people engaged in a common purpose (the *subject*: in this case the teachers), the direction of their work (the *object* or motive: in this case the mathematical learning of their students), the *goal-directed actions* which are needed to achieve the object, and the *operations*, or routines, which keep the system working fluently (Leont'ev, 1974; Nardi, 1996). These operations can be subcategorised as *rules*, *community* characteristics, and *division of labour*. These features are in dialectical relation, so that if one changes, other changes have to take place to adjust the whole system, sometimes leading to collapse, other times generating new more resilient structures (Williams, Davis & Black, 2007 p. 3). This inherent instability is recognition of the nature of human agency within a system, and that the object is dependent on how it is understood by the people concerned. Williams, Davis and Black (2007) point out the subtle nature of 'object' as a material embodiment of a collective purpose (p. 3) but we found this hard to operationalise and chose instead to conflate the ideas of object and desired outcome, described separately by Engestrom (1987), and see the teachers' stated purpose as the object.

The capacity of activity theory to describe the interplay between stable practices and instability in the departments suggested it would be a suitable frame for our analysis. During the analysis we noticed that the *object* of the system, students' learning, was in the same sense the object of individual classrooms, and that classrooms and departments could be seen as interacting activity systems with the same object, albeit with different subjects and communities. Third generation activity theory attempts to describe such interacting systems and Engeström (1998) conjoined two systems, one being classroom activity and one being staffroom activity, to analyse the behaviour of a school mathematics department undergoing deliberate change. However, in his analysis the change itself was the object of the activity, whereas in ours the object was to improve students' learning. Venkatakrishnan used activity theory to explore how school mathematics departments respond differently to externally imposed change, and her main interacting systems were school departments and local authority support structures. Having analysed this interaction over time, she found that different appropriation of tools disrupted assumptions about how far the objects of activity were shared (Venkatakrishnan, 2005). The capacity of activity theory to pinpoint differences in tool-use as a significant contradictory element in interactions between systems seemed powerful, which is why we adopted it. The role of *mediating tools* in activity is multi-layered. Within mathematics departments, where the main purpose is the intentional teaching of mathematics to others, some tools are obvious - textbooks, shared schemes of work, shared resources, curriculum and assessment guidelines – but other tools that relate to the learning of the system itself are less obvious. In our data collection and analysis we were looking for changes in the department activity, such as rules, object and tool-use, over time.

We were also influenced by its use to unravel other aspects of mathematics education, such as its capacity to explain students' mathematical actions in situations (Williams, Linchevski & Kutscher, 2008), and its descriptive power for mathematics classrooms (Jaworski & Potari, 2009).

2. Complexity

Davis and Simmt (2003) explain how complexity theory has developed in recognition of the fact that some systems cannot be understood using analytic tools which separate components. That is, the behaviour of some systems cannot be predicted by analysing the actions of individual elements of the system. This is not simply a problem related to the difficulty of analysing large numbers of interactions between components but to qualitative differences between systems that are complicated by virtue of the numbers of interactions, and systems that are complex. Complex systems typically comprise living agents who are autonomous, at least to some extent, and are characterised by features that are emergent in that they arise from the interactions of agents but cannot be directly attributed to particular agents (Davis & Simmt, 2003).

Complex systems are also adaptive in that their response to a given stimulus is dependent not only on the stimulus but on the history of the system. Complex systems thus embody their histories as they adapt to their environment and hence can be described as learning. Applied to human systems, learning can be seen as an emergent feature of the collective, and knowledge as residing with the collective rather than with individuals (Davis & Simmt, 2003). This is not to deny the existence of individual learning because individuals too can be described as complex systems nested within others. Indeed, Davis and Simmt (2003) illustrated the nestedness of complex systems by referring to the relationships between cells, organs, individuals, and society, all of which learn in the sense of adapting to their environments. Like them, however, for this paper we are looking only at one level, that of a collective of individuals although where necessary we acknowledge the impacts of aspects of the broader system within which the department was nested and hence operated.

Davis and colleagues (e.g., Davis, 2004; Davis & Simmt, 2003; Davis & Sumara, 2005) have described educational settings in terms of complexity theory and have proposed five necessary, but not sufficient, conditions for emergence to occur. These are: *diversity* among agents (typically students in a class) which allows for novel responses; *redundancy* in the sense that agents have sufficient in common to allow meaningful interaction and to compensate for each other's weaknesses; *enabling constraints* that balance order and focus in the collective's activity with the expression of its diversity; *decentralised control* that recognises that outcomes, including the emergence of complexity, cannot be predicted but instead emerge from the collective activities of agents; and *neighbour interactions* between ideas rather than simply between agents.

Although these conditions have proved useful in describing educational settings (e.g., Sinclair, 2004) for classrooms the conditions have to allow for intentionality on the part of a teacher (Towers & Davis, 2002). Davis (2005) attempts to deal with the dual role of the teacher as one of many agents in a classroom in which purpose is an emergent feature, and the teacher's intentionality by likening the teacher to the 'consciousness of the collective' whose role is to direct and focus attention and to choose among possible interpretations and actions open to the collective. Similarly Osberg and Biesta (2008) identified a tension between leaving open possibilities for emergence and acting in ways designed, but in no way guaranteed, to achieve desired ends thereby necessarily closing down alternate possibilities. Although helpful, this falls short of recognising the capacity for intentionality characteristic of *all* agents in a collective of human beings. A skilled leader is able to notice emerging patterns, intervene to stabilise those that are helpful (in terms of his/her intentions) and destabilise those that are not, and to structure the environment by *seeding* it or creating *attractors* around which patterns of interaction emerge, so that desired purposes and outcomes are likely to emerge (Kurtz & Snowden, 2003; Cunningham, 2004). Cunningham shows how a skilled mathematics educator can 'lead' a department into change by constructing new attractors which emphasise emergent patterns (2004). In our analysis we focus on department life as enacted in the mathematics staffroom, where the head of department (HoD) as leader might act as the 'consciousness of the collective' by managing the enabling constraints and providing attractors.

3. Study A

Study A was a three-year funded ethnographic study designed to tell the story of three mathematics departments as they set about making significant changes to the ways in which they teach mathematics to low-attaining students. Their decisions to make change arose internally, and we researched aspects of this process, joining them at the start of their initiatives. In Engestrom's terms, they were already undergoing 'expansive transformation' when 'the object and motive of the activity are reconceptualized to embrace a radically wider horizon of possibilities than in the previous mode of the activity' (2001, p. 137).

We hoped to relate the departments' reconceptualised activity to the achievement of the students in one cohort, to identify factors which contributed to success or otherwise, and to tell plausible stories about how the departments operated. For this paper only a part of this study is used in which the departments, organising the mathematical experiences of students, are the units of analysis[1]. Although other data were collected for the broader study we focus here on teachers' reports collected through semi-structured interviews which were undertaken three times during the study. In all interviews we asked some

[1] For more information about the study see www.cmtp.co.uk

informational questions and some more open questions designed to get them to talk about their own perspectives and experiences. For example, in the second interview the open questions included:

- What have your priorities been for year 7 teaching during this year? Have these changed during the year?
- Has your teaching changed during the year?
- What have been the main difficulties/successes in teaching year 7?
- Have you made any input into planning, apart from your own lessons?
- We work as a team to ……………………………….
- We work individually to ……………………………….

Interview data is subjective but is appropriate for analysis using an activity theory framework because activity systems depend on human consciousness and agency. We reasoned that we would learn about agreements, disagreements, and what actually happened through interviews rather than by looking at systemic artefacts such as school schemes of work and assessment regimes. The data were analysed by identifying and categorising content according to the features of activity theory; thus we categorised what was said about subjects, objects, tools, rules, community and division of labour[2]. These categories enabled us to sort what was said in each separate interview, and enabled comparisons. In this way we learnt how teachers saw the object of the activity, what took the form of rules, how labour was divided, and how other people were connected to the activity. We heard them talk explicitly about resources, but also noticed what else they said they used to help them in their work. For each teacher we developed two versions for each interview of the outline triangle in Figure 1, separating what they said about their own classrooms, where they had authority, from what they said about the department, where they were one of a collective, and then compared the contents of each of the six vertices. We then collated the contents and the comparisons for each school to get a picture of how teachers perceived their activity as a department, and what changes were apparent during the study. Between-teacher comparisons let us identify shared understandings and contradictions within schools, and similarities and differences between schools. Comparisons over time helped us to see features of change.

Figure 1. The work of the mathematics departments seen from an activity theory perspective (after Engestrom, 1998)

The multi-layered process of analysis and comparison threw up many interesting observations, even before comparisons were carried out. Having decided that activity theory was an appropriate framework, what followed was an exercise in: fitting the data to the structure, and seeing what did not fit; seeing whether the structure could be interpreted to accommodate the data; and questioning both the structure and the data. The analytical questions are: 'What can this data tell me if I look at it with this perspective?' and 'What do I learn about this perspective from this data?' The triangle in Figure 1 gives more detail about where the interview contents were interpreted and structured in our analysis, and the headings on the connecting lines indicate how teachers talked about relationships between

[2] Jaworski and Potari (2009) give more detail about the role of these categories in activity theory

the categories[3]. We shall now give examples of categories that arose in teachers' reports that illustrate how we had to adapt the frame to take account of what we were told:

Accountability: One feature of English mathematics departments is that external and institutional processes of accountability impact strongly on departments' and individual teachers' autonomy. In all three schools, teachers were accountable to outsiders, but within departments accountability was differently interpreted by different teachers. The guidance given by HoDs was seen as prescriptive by several teachers, although it was not intended to be so. For this reason the idea of accountability does not appear under 'rules' or 'community' as one might expect, but edges more towards individual interpretation of the object, while the 'systems of account' relate individual teachers to the community through how they think they are expected to behave.

Resources: Many teachers talked of contributing ideas to the department resource bank in their school. On the face of it this would describe a division of labour. However, by contributing an idea to the bank, they were also contributing *their ways of seeing the teaching of mathematics*, either through the bank or through discussions about their suggestions. Thus, their knowledge was more than something they did individually, but became available to be used by others – a potential pedagogical tool. In this sense, the ways of communication of individual knowledge act as mediating tools to affect learning about pedagogy. This description of individual knowledge acting as a tool within a department, to be taken up and used by others, fits with how teachers talked about it, picking it up and taking it with them into their classrooms, than to see it as part of more generally distributed knowledge which cannot be accessed from the classroom[4].

Meetings: Department meetings could be described either as a feature of the way the community operates, or as part of the rule-structure of the department, but the discussions which take place in them can be seen as mediating pedagogical learning. In many meetings, specific mathematical tasks were undertaken and discussed. When interviewees mentioned meetings it was always in the sense of resource, rather than in the sense of a departmental structure or rules of behaviour. This contrasts with other kinds of subject meetings in schools which can be administrative, organisational or coercive in tone.

Division of labour: There were interesting differences between what people said was supposed to happen and what actually happened within the department. The most common was that they were all supposed to contribute ideas, but in the schools where this meant 'put some lesson plans into the file' most claimed not to have done that. Thus 'division of labour' was that some did and some did not populate the resource file, whereas 'rules' included the expectation that all would do so. We expanded 'rules' to include 'expectations' so that 'division of labour' could be left to describe what actually happened rather than what was supposed to happen.

Comparing the contents of the triangular layout over time revealed some significant aspects of change, which were corroborated with other data from the system: HoD reports, documents and meeting observations. Most systemic renegotiations, apart from the initial deliberate changes made before our study began, took place during the first eighteen months. There were significant changes in object, tools, division of labour and rules in all three schools but here we will report only on how the object changed. Asked about priorities for year 7, the teachers in one of the schools began the year with a shared aim,

[3] Literature about this triangle is copious and cannot be summarised here, see Engestrom 1998. What is new is the use of the connecting lines to make sense of how individuals interpret aspects of the activity.

[4] We recognise the danger, pointed out by one reviewer, of extending the meaning of 'tool' to embrace knowledge but found no other way, within this theory, to take full account of how teachers used the word.

articulated by all teachers, that students should ‘enjoy’ mathematics. This appeared to have been adopted as a proxy for ‘improving learning’ for year 7, by way of establishing new working practices and attitudes towards mathematics. By the end of the year many teachers were saying that they were concerned about students’ basic knowledge and that ‘skills’ were one of their priorities. This was not a stated aim through departmental communication channels, but had emerged from the grounded experience of the teachers. For Engeström (1998), the interesting thing about systems is how they learn, where learning is understood as the constant flux between internal inconsistencies arising partly from individuals; “... multi-voicedness is multiplied in networks of interacting activity systems. It is a source of trouble and a source of innovation, demanding actions of translation and negotiation.” (Engestrom 2001 p.136). Concern about ‘basics’ was initially a rupture between the department object and the objects of individual classrooms, but this was followed by transformation of the object through restructuring a tool, the scheme of work, and renegotiating priorities. Both of these happened explicitly and collectively; individuals’ ways of seeing their own work changed privately in planning and classroom teaching. After eighteen months, no major systemic changes took place – we could say that their activity was relatively stable.

Comparing teachers revealed a wide diversity of interpretations, priorities, views and reported actions. These are reported fully elsewhere[5] but for the purposes of this paper we focus on one feature, that of marginalisation. Many kinds of diversity continued throughout the project between all teachers, particularly at the level of individual classrooms and lessons, and did not appear to create conflict within the system. Other differences which arose from several teachers, such as the concerns about ‘basics’ did lead to change. Overt differences were seen by HoDs to be training needs. They talked to us of ‘believers and unbelievers’ or ‘teachers who are on board or not’. We noticed covert different interpretations when teachers acted as if they were talking about the same thing. In one school, some teachers talked about improving learning by using open-ended tasks and investigating mathematics while the HoD talked about improving learning by focusing on mathematical structures. Meanwhile, in both formal and informal interactions, everyone appeared to believe they were talking about the same thing apart from a few teachers who were known to be adhering to a traditional repetitive form of teaching. We saw this as differences in interpretation of the shared object. For some teachers this difference was shown in the very different uses they made of ‘the same’ artefacts, such as particular worksheets, or written tasks, or manipulables. The mathematical meanings with which they were imbued by individual teachers in classrooms were different, and knowledge of pedagogy was not unambiguously mediated through these resources. A few teachers did not use the resource bank at all: so the resource bank was not a common tool, although the teachers were actors in what purported to be the same system. Instead some teachers imported tools and meanings from outside, in one instance with the declared aim to teach in ways that conflicted with department expectations, in another because the teacher was institutionally marginalised. Gradually we saw that there was such variety of tool use and object, often masked by the common use of language, that the word ‘system’ might not include all mathematics teachers. A key feature of marginalisation was that these teachers did not refer to each other’s knowledge or department meetings in the tool-like way that other teachers did. Over time, we recognised that it was these resources, rather than the material banks, that functioned as tools for change, and teachers who did not use them

[5] www.cmtp.co.uk

were not fully engaged with changes in the system. We identified four kinds of marginalisation which placed some teachers doing the same overt work, in the same team, outside the activity system as described above:

self-marginalisation as a result of unwillingness to change: these teachers were identifiable through what they said to us, their use of different tools, or different use of the same tools, or even through their refusal to agree to participate in our research;

institutional marginalisation due to timetabling, conflicting priorities, or part-time employment: these teachers were identifiable from documentary evidence and absence in the team room; they might try to use the same tools, but their interpretation might be different;

ideological marginalisation when a teacher disagrees with the prevailing values and policies but nevertheless continues to work in accordance with the department, perhaps adopting new tools with only superficial understanding: some of these teachers were self-identified in interviews, others by our observations of their very different enactments;

epistemological marginalisation, in which a teacher's mathematical knowledge is too weak or different to understand department discussions; these were identifiable in department meetings, particularly during the mathematical tasks.

Activity theory helped us identify important features of departmental activity and how they inter-relate. We have been able to represent nearly all shared aspects of department activity as described by the teachers. We were able to describe systemic influences on relationships between the points on the triangle by seeing different qualities in these relationships for different schools as reported to us by teachers. These gave rise to labels on the connecting lines of the triangle. From these linkages, and attempts at linkage, we have described some conflicting aspects which had the potential to change the systems. This analysis did not, however, enable us to make sense of different teachers' interpretations of goals and artefacts in their action, and how these related to the department's work. We saw evidence of their different interpretations in their teaching and their interviews, but not how these differences influenced departmental activity, except through marginalisation. Nor did it enable us to track the ruptures which depended on *interpretations* of the object (what it means for students to learn more mathematics) rather than changes in the stated object itself, because teachers could be talking the same language but enacting this differently in their own classrooms – places where other teachers were not affected. Indeed, activity theory did not allow us to understand fully if and how individual interpretations affected the system – but it did reveal them, and showed that some differences were conflicting and that there were splits and potential splits, both known about and unknown. However, there were other individual differences in interpretation that were coexistent, often among core team members, for which no negotiation or reconceptualisation appeared to be necessary. Activity theory, while alerting us to marginalisation, changes of object, and the difference between tools for department change and tools for normal maintenance of mathematics teaching, did not illuminate the 'normal' diversity of the systems which was apparent when core activity appeared to be relatively stable.

4. Study B

Study B concerned one department which had also recently undergone significant change, in this case a change of staff. There was a new HoD, a new teacher with responsibility for Key Stage 3 (lower secondary) and essentially ‘third in department’, and two newly qualified teachers (NQTs). The school had specialist mathematics status, and the extra funding which derived from this meant that the HoD had been appointed with a brief which included teacher development, community engagement, and dissemination of good practice. Because this study was designed to ‘test’ the use of complexity theory an initial description of diversity within the team is taken as the starting point. The existing team comprised six teachers, including four teachers who had whole school management responsibilities and taught less than the full load. Two of the teachers who held other responsibilities were not mathematics specialists but had trained in physical education and music. Both had taught mathematics for many years and were well-established members of the mathematics department. The other teachers all had strong backgrounds in mathematics. Three of the team had recently been engaged in academic professional development courses in mathematics teaching at a nearby university.

The study was conducted in the first term of the school year and aimed to describe how the department developed. Although the intention was to use complexity theory for this purpose this was necessarily a tentative decision until emergent features were evident, since it is the occurrence of emergence that defines a complex system. Particular foci were the development of shared beliefs and the ways in which individuals adapted to one another and influenced the department as a whole. Data comprised: individual interviews with each of the ten department members at the beginning and end of the term; additional interviews with the HoD, the new third in department, a newly qualified teacher, and a teacher who had been at the school for a number of years; and audio-tapes and observations of departmental meetings.

Complexity theory was considered an appropriate theoretical tool in this context to learn more about the department because the new HoD’s brief included change and learning at the departmental level. In addition, although an established department may have norms of practice and interaction that have been implicitly or explicitly agreed in the past, the influx of new staff necessarily required the renegotiation of roles, relationships, procedures, and new patterns of interaction whose outcomes would be unpredictable making it a setting in which complex emergence would be likely.

Data analysis consisted of reading and re-reading the transcripts and categorising, in a grounded way, what the teachers said about their work. We had asked them about the aims of their teaching, factors that influenced these, the ethos of the department, their perceptions of their own place in and contribution to the department, ways in which they believed the department had or was changing, and the kinds of activities and interactions that occurred in their day-to-day experience of working in the department. The categorised data were then examined for shifts in the thinking of individuals, and commonalities and diversity between the beliefs and perceptions of individuals. It was apparent that over time there were shifts in the ways in which teachers articulated their views including increased use of the same or similar phrases which were sometimes but not always attributed to another staff member, most commonly the HoD. So far the data collection is similar to that used in Study A, but the search is for emergent phenomena and shifts in thinking, rather than particular categories of the system. Shifts of thinking are seen as elements of the data, rather than differences between elements of the data.

Emergent phenomena were identified as common themes and included:

- an increasingly shared understanding of the meaning and importance of mathematical thinking in improving students’ attainment

- consensus around the idea of providing access to higher levels of attainment for all students
- a long term view of improving attainment
- a shared sense that the department was supportive.

Although it was possible to identify contributions made to each of these by individuals their emergence is not entirely explicable in terms of direct influence from individuals, even those in positions of leadership. Rather, they appeared to arise from interactions among the teachers in a form that was not precisely represented by any individual contribution. It was possible to identify in the data particular factors that contributed to existence in the department of each of the five conditions for complex emergence. Examples are provided in the discussion that follows.

The particular focus in this paper is the use of complexity theory to analyse retrospectively the HoD's attempts to influence mathematics teaching practices in the department. Since emergent phenomena can be perceived but not predicted (Kurtz & Snowden, 2003) such retrospectivity would have been necessary even if she had been consciously attempting to create the conditions for complexity (Davis & Simmt, 2003). Evidence of the extent to which each of the five conditions for complexity were present in the department and the purposeful use and management of attractors by the HoD are described below. Complexity theory orientates us to look for attractors and this focuses on leader sensitivities, where in study A the analysis of change of activity did not direct us to particular aspects of the HoD role.

The HoD in study B had clear purposes in mind which she articulated throughout the term in the context of interviews, staff meetings, and in informal contexts. These related to enhancing students' opportunities to achieve, and focussing on students' thinking and how that could be moved forward in such a way that they achieved deep understanding of mathematical structures. She saw the two as related in that deep thinking and understanding would contribute to long term gains in achievement. She explicitly likened the department's learning to that of students and compared the way she would like the department to operate to the way in which she wanted classes to operate - that is, characterised by deep, independent thinking, sharing of perspectives, and both individual and collective construction of understanding.

The ingredients for complex emergence, (diversity, redundancy, enabling constraints, decentralised control and neighbour interactions), appear to have been present in the department partly as a result of the HoD's choices and partly as a result of outside influences upon it. The *diversity* of views and approaches to mathematics teaching represented by the ten teachers was mentioned by several teachers when prompted to describe the department's strengths (rather than the differences and changes we asked about in study A). The HoD also acknowledged the diversity represented by the teachers when she described the professional learning needs of the department as follows:

> … it's a question of people really building up their own areas of expertise and following those rather than one size fits all. In terms of one size fits all that's more of our working together rather than using people from outside. Take for instance, how to introduce algebra, I think we've got the skills between us to work together on that, ...

Much of the *redundancy* evident was a consequence of the teachers' familiarity with broad understandings of the mathematics in the English National Curriculum, examination procedures, and usual school organisational practices. The importance of ensuring that the examination results were satisfactory was taken as a given. Another source of redundancy was the strong mathematics background of eight of the teachers, and extensive experience of mathematics teaching of all except the NQTs, enabled all to participate in conversations of a mathematical nature. For example, on one occasion teachers were asked to bring examples of how they had incorporated the idea of equivalence into their mathematics teaching of any topic with any class. The request included a brainstorm of opportunities in which the idea might arise. All of the teachers participated in the initial discussion and, in a subsequent meeting, most teachers did report examples of highlighting equivalence in their teaching. Importantly, all of the examples offered were accepted.

Interestingly, the externally imposed constraints of curriculum and examinations not only contributed to redundancy but also appeared, by virtue of their familiarity, to act as *enabling constraints* for some teachers. It seemed that the system requirements had been internalised by all of the experienced teachers to such an extent that they felt some degree of freedom to experiment with teaching approaches. The HoD expressed a similar view of school level policies, explaining that, "We really do have quite a lot of freedom, that's the sort of feeling I have". The episode relating to the mathematical concept of equivalence also illustrates how for some teachers being provided with a specific task and expected to report back was enabling.

Several teachers referred to the episode in subsequent interviews and it became part of the redundancy that facilitated further interaction while simultaneously reinforcing the value that teachers attached to their diversity.

Enabling constraints were similarly provided by the HoD as she worked to encourage conversations about students' thinking. These included asking teachers to bring examples of students' books to a departmental meeting so that the ways of providing feedback could be discussed. Initially only the HoD herself had examples to share but at a subsequent meeting a few other teachers also brought examples. The purpose of enabling constraints is to balance order and the expression of diversity (Davis & Simmt, 2003) but, since the unit of analysis is the system as a whole, complexity theory does not offer an explanation of why the same constraints appear to be enabling for some individuals but to limit or obstruct others.

From Kurtz and Snowden's (2003) perspective, enabling constraints can be thought of as *attractors* which establish a degree of order around them. The unpredictability of the impact or effectiveness of attractors, or even whether an influence on a system acts as an attractor at all, is inherent in the nature of complex systems (Kurtz & Snowden, 2003).

Other attractors included the HoD's enthusiasm for mathematics and for teaching, her constant references to students' thinking and the need to move it forward, and the fact that most of the teachers in the department had desk space in a team room. The HoD's references to thinking included an A4 poster she created with the slogan, "Learning to Think, Thinking to Learn" that was displayed in several of the mathematics classrooms and the team room, and was referred to by several teachers when they were asked about the department's ethos. The energy that the HoD devoted to teaching was evident to her colleagues who saw her as having high standards of effort.

The team room's function as an attractor was due to its role in facilitating *neighbour interactions*. The HoD, the two NQTs, the new 'second in charge', and two teachers who had been in the school for a number of years all spent most of their non-teaching time in that space and informally shared their practice. The usefulness of these conversations was described by the HoD as follows:

> Sometimes we're working and talking at the same time, there's lots of it, and somebody else comes in and they join in. People seem to be much more ready for that than if you were to convene another formal meeting because they don't feel they have to be there, they're drawn in by interest, …

Others who did not work in the team room because they had office space elsewhere or who chose to work in their classrooms still made regular visits to the room to collect and return resources stored there or to seek out advice. The HoD recognised the value of such interaction and, in Kurtz and Snowden's (2003) terms, acted to stabilise this emergent pattern by proactively ensuring that she regularly visited the teachers who primarily worked elsewhere. The presence of biscuits, tea and coffee helped, and we saw several copies of a popularising mathematics book scattered around the room.

The department was necessarily constrained by school and system requirements but in other ways the teachers were autonomous and hence control was largely *decentralised*. The HoD was aware of the need to provide a safe environment in which people could take risks as they tried to change their practice. To this end she avoided directly observing her colleagues' teaching but instead monitored practice principally through conversations with them and also by listening to classes as she walked through the corridors. In her words:

> I'm not keen on doing things which I think leave the person feeling insecure and on the hop. What I want to do is … get somebody to take risks and work outside their comfort zone. They're much less likely to do that if they think you're about to barge in any second … I probably do a bit more from the corridor than people realise I do.

This department illustrates well the inter-dependence of the conditions for complex emergence. Figure 2 captures diagrammatically something of the nature of these connections. Complex emergence depends upon the agents in a system being diverse and yet with sufficient in common to allow for meaningful interaction in the sense of sharing ideas. In Figure 2 diversity encompasses all of the knowledge and experience present in the system. It is necessarily in a constant state of flux as agents change in response to their experiences including of each other. A subset of that combined experience is shared and it also is constantly changing as well as different elements being held in common in relation to specific agents. The dotted lined surrounding the enclosed circle is intended to show the interplay, facilitated by neighbour interactions, between the system's diversity and redundancy. The extent and impact of interactions depends upon the extent and nature of enabling constraints and decentralised control. Rather than being thought of as in tension with one another, both can facilitate neighbour interactions, with decentralised control allowing agents to exercise autonomy and enabling constraints filling a similar role by providing safe boundaries thereby encouraging the expression of diversity. Together they provide an environment in which individuals can express their diverse ideas and have them, to borrow a term from Davis and Simmt (2003), collide. In attempting to change the department, the HoD's aim was to build a shared vision of mathematics teaching and hence to increase redundancy in the system. At the same time all of the teachers valued their diversity which was simultaneously encouraged. Sharing ideas and experiences did not simply amount to transferring ideas from the diversity region to the redundancy circle, but rather at least part of the redundancy created could be considered emergent in that it comprised new understandings that resulted from the interaction of the group's diverse ideas and experiences but was not simply the sum of them.

Figure 2: Relationships among conditions for complex emergence

Finally, this analysis displays aspects which we assumed may be especially pertinent for mathematics departments: the HoD's intentionality in supporting the emergence of a learning environment reflects the high turnover of staff and their diverse contributions; the political and institutional constraints are enabling; the shared epistemic context is one of several components which ensure redundancy and what emerges is concerned with particularly mathematical ideas about students' learning.

5. Comparing the affordances of the different theoretical perspectives

The overarching question in choosing between complexity theory and activity theory is: 'is it more revealing for our purposes to characterise this department according to diversity and emergence or to look on it as structured by the means of mediation and the appropriation of tools?' There are three sources of difference: the theories, the departments, and how our use of the theories led us to focus on and observe different features. We are interested in the last of these sources. The key differences in this regard concern at the outset different ways in which the two perspectives orient data collection; assuming the presence of particular features and looking for changes to and conflicts among them in activity theory compared to complexity theory's assumption of nothing other than the inherence and unpredictability of change that allows the direction of data collection to evolve as features emerge. Implicit in these orientations are contrasting views of change as emergent without clear causality in complexity theory compared to disruptive and framed as conflict in activity theory. A third difference relates to the ways in which the two perspectives situate a given level of analysis, such as a department, within its broader context. Whereas activity theory views external forces as potentially disruptive and conflicting, complexity theory regards any level of analysis as one of many agents comprising another potentially complex system in which it is nested and itself being constituted of complex systems nested within it, each of which is unpredictable and able to give rise to emergent features that influence the whole in ways regarded as natural rather than disruptive.

In all the departments considered in these studies, there were aspects of their functioning that were known, predictable, and governed by agreed procedures and allocated responsibilities. In Study B these aspects included the compliance with examination entry procedures and setting, but the aim of improving students' attainment was a shared goal in relation to which each teacher acted autonomously, albeit influenced by their interactions with one another and particularly by the intentions of the HoD, which included the encouragement of diverse practices. The HoD had a very clear vision of the direction of change that she wanted, and a strong personal agenda for the nature of change, but the ultimate aim of maximising attainment for all was uncontentious – it was the means of achieving this that held diversity. In Study A important aspects of the departments' efforts to achieve their aim were much more structured within the departments themselves. The aims of the departments were tightly defined as relating to improving the learning of previously low attaining students, and our research was to focus on the conceptualisation and enactment of this aim. The aims were subject to timelines and measurement, and were not necessarily in tune with the aims of each individual within the system. For this reason they were managed centrally with questions like, 'Who will take responsibility for this necessary task or role?' (division of labour) and, 'What common tools do we need to carry this out?' Choices made by leaders in relation to bringing about change, particularly whether they attempt to facilitate the emergence of aims, or seek to devise and impose systems that will further an aim, are highly relevant to whether one thinks of the system as complex or as activity. Study A illustrates that simply having an *object* around which other aspects of the activity are appropriately aligned does not guarantee its maintenance and achievement. In study A we analysed the elements of the system to identify their intended interactions and effects, whereas in B we began from the complexity theory premise that this is futile and looked for how the system was influenced to occasion the emergence of desired outcomes.

Both theories acknowledge the inherent dynamism and unpredictability of the enterprise and the need for ongoing adjustment. Activity theory, however, sees change as sequential structural disruption whose roots are often foreseeable, in that systems necessarily contain, within their ways of functioning, relationships which might break down, or might be in conflict with other relationships. Thus change is manifested as reorientation of parts of the system; renegotiation of roles and rules; introduction of new mediating tools and meanings; and redefinition of objects. Activity theory predicts and models the reorganisation which precedes and follows a change in aims, and also shows up the potential problems arising from a lack of shared objects, or from contradictory interpretations of objects. Complexity theory embraces change as a necessary characteristic of systems, often unforeseeable, and sees 'adjusting' as part of the overall dynamic functioning of the system. Complexity theory describes fluid systems in which related members are able to take a large number of autonomous decisions (decentralised control), members work in parallel and might influence each other through neighbourhood. There is no sense of 'preceding' or 'following' change, since change is always taking place. Activity theory identifies possible causes of change, so that a leader might introduce new tools or new rules to initiate changes in activity; complexity theory avoids assumptions of causality, and offers instead the idea of attractors around which new events accrete. In study A we saw the introduction new aims and resources at the start of the change process, and the subsequent changes in these; in study B we saw the management of attractors.

The conflict orientation of activity theory led us to incorporate some institutional requirements directly as rules, and to look for outside influences as power vectors, which may or may not have been benign in relation to department aims. Incorporating institutional and outside requirements in Study B as aspects of complexity did not show whether they had an alien, contradictory, quality or were enabling. Complexity theory treats these as another complex system within which departments are nested. It is notable that this study took place at a time when English school mathematics teaching operated within detailed, imposed, prescription. The HoD who created an autonomous system in study B had to be courageous to do so, but whereas activity theory would illuminate this because of potential structural conflicts between interacting systems, complexity theory does not.

In using these theories, our treatment of diversity was different. In study B diversity was assumed and we looked for how this was coordinated and enhanced through, for example, neighbour interactions. In study A, we sought differences. This led us to identify types of marginalisation and see how this related to use of tools of change. However, this was not merely a difference of research perspective – the HoDs in study A talked of difference as a problem and trying, through training and co-planning, to generate more alignment of practice where the HoD in study B encouraged diversity.

5.1 Mathematics departments

Much that we have written could apply to any kind of subject department in schools, yet at the start of the paper we included the epistemic context as a justification of our focus. We were interested to know if activity theory and complexity theory were differently informative about the mathematical practices of the departments, apart from offering a way to make sense of generic aspects and thus lay bare the heart of mathematical activity.

In Study A the analysis suggested that personal mathematical and didactic knowledge were seen to have tool-like roles by other teachers, in that non-marginalised teachers drew on and used each others' knowledge as resources. This is in addition to the role teachers' mathematical knowledge has in informing rules and expectations for classroom activity, and in contributing to material resource banks. Not everyone's knowledge was drawn on in this manner however. In one department, a senior member took on himself the role of introducing mathematical tasks in meetings. These tasks were designed to draw attention to particular aspects of mathematical understanding, in an attempt to generate discussion not just of the machinery of teaching but of the underlying mathematical ideas. For example, one task was designed to generate several ways to represent trigonometric ratios, with the intention that teachers could then discuss which they might use to develop deep understanding with students. There was little discussion, and teachers were more subdued than when discussing future planning or past lessons. The activity revealed that one teacher had a weak understanding of algebra, in conventional terms, as she believed that labelling the vertices with letters was itself an algebraic move. Neither the knowledge about the differentiated power of representations, introduced in the task, nor the knowledge of the teacher who had that particular view of algebra, were drawn on further[6].

It would be possible to analyse this episode in terms of power relationships, but it is also possible to see the community as depending on acting as if everyone had similar mathematical knowledge, a kind of shared middle, rather than anyone being less or more knowledgeable, and this event as disrupting that view. Other evidence that the overt rhetoric about shared knowledge did not tell the whole story was that in one department which had a policy of sharing resources, very few teachers used the central bank but all made *ad hoc* arrangements between teachers. The other two departments also worked on mathematics from time to time, but in a different way: tasks were introduced by a range of teachers and discussed in terms of their classroom use – these were not special tasks for the teachers' learning. However, in one school teachers engaged with the task themselves before discussing its use, while in the other it was more likely that the introducing teacher would say where the task could 'be taken', but without discussion of how it could 'be taken' there or how such 'taking' related to students' learning. The focus on classroom tasks avoided forced public revelations about different knowledge, or mathematically incorrect statements, yet teachers working in self-chosen pairs did seem willing to declare that they 'didn't know' some of the mathematics.

While nearly all teachers made connections between mathematical ideas for pedagogic purposes, there were also a few instances we observed in which teachers had explored mathematical ideas and reported their explorations, as mathematicians might, for aesthetic reasons or to express excitement. There was some mirroring of what Burton (2002) described as the culture of mathematicians, but this was nearly always contextualised in discussions about pedagogy and classroom tasks

Similarly in Study B, knowledge of a certain unspecified level of mathematical knowledge was assumed. One teacher was open about having less mathematical knowledge than others and gave it as a reason for not being comfortable teaching the higher sets. Nevertheless his knowledge was sufficient to enable participation in mathematical discussions in department meetings. Rather than being divisive this difference was regarded as illustrative of the diversity in the department that all valued. Indeed, the department's embrace of diversity may have enabled this personal discomfort

[6] Some details have been changed in this episode for ethical reasons.

to be expressed in the presence of the researcher, and complexity theory provided the tools for us to notice it, whereas in Study A we were unaware of anyone expressing their discomfort in public, and no one expressed it to us.

Complexity theory tells us that diversity and unpredictability are inherent in human systems and suggests mechanisms for leaders to occasion emergent phenomena, whereas activity theory identifies potential conflicts in activities that at least for a time seem structured or can usefully be treated as if they are, and hence predictable. While both recognise that individual difference is expected, for example differences in tool appropriation, neither seems capable of adequately incorporating individual differences of action and interpretation within the system, neither is adequate for accounting for individual knowledge, and neither explains different individual reactions to, for example, constraints. Most importantly for mathematics teaching, our use of neither theory gave us a way to explore the changes in the ways in which teachers handled mathematical content – changes that would affect achievement of the aims. In complexity theory this might necessitate analysis of the mathematical content of neighbour interactions and the nesting of classrooms in departments, schools and systems; in activity theory this might involve analysing the mathematical affordances of tools.

Our findings are consistent with those of Even and Schwarz (2003) in that the two theoretical perspectives did ask different questions and explain similar phenomena differently. However, we have tried to show that the different affordances of activity theory and complexity theory for understanding departments are much deeper and broader in scope amounting to different views of the world. Whereas activity theory identifies key aspects of a department's functioning (its object, subjects, community, tools, rules and division of labour) and tracks changes in each of these over time as the various elements interact, complexity does not assume that any of these elements exist at the outset or indeed ever will. In our studies, perhaps as a consequence of the fact that both involved mathematics departments, many analogous features were noticed. The key difference is that activity theory pointed us to look for features that fit the categories that define the system, whereas complexity prompted us to look, through the course of the study, for new features that developed over time without clear causal lineage and to seek evidence of conditions within the system that appeared to facilitate their emergence.

Acknowledgements

Study A was funded by the Esmee Fairbairn Foundation, grant ED 05-1638.

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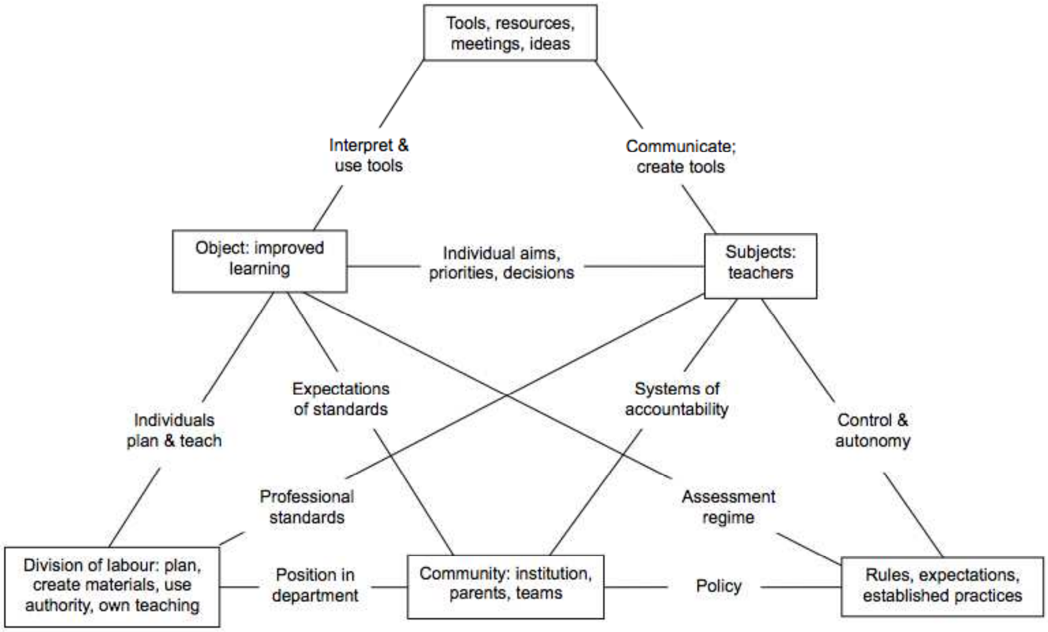

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