**Sequence 4**

Start a new construction with a new name.

Draw triangle ABC

Construct the orthocentre (label it H) and the circumcentre (label it O) and the **median point** (label it M): intersection of lines from each vertex to mid-point of the opposite side. Hide all the construction lines so you can see what is going on.

Join HO with a line (not a line segment). OMH is a straight line with M trisecting OH.

Wiggle ABC about a bit**.** Is what I have just said still true? Convince?

OMH is called the **Euler line.**

[Dick’s proof is in the proof notes and I don’t yet (Feb 2021) understand it completely. However, because of the constructions you have done so far, and assuming that you know the median point is one-third of the median line segment, you might be able to show that the points O,M,H do indeed form a straight line trisected by M. Be careful not to assume they are in a straight line – I had to draw a pencil diagram in which they did not look like a straight line.]

FInd the mid-point of OH and label it N (a label you have seen before). You might like to change to a joyous colour. Draw a circle with centre N and radius its distance to the mid-point of a side. You have seen this circle before.

Join H to the vertices A,B and C and mark the mid-points of HA, HB, and HC. The circle you have drawn is famously known as the **nine-points circle.**

Look back at the previous construction file: does the medial circle go through the mid-points of the line segments from the orthocentre to the vertices?

What’s the same? What’s different (don’t spend too long on the ‘different’ question!)

Save your work and start a new file for the next sequence.

**Leeman-Gossard theorem**

The Euler line of a triangle ABC meets the sides at X,Y and Z. The Euler lines of triangles AYZ, BZX, CXY (maybe use three different colours to construct them) are parallel to the respective sides of ABC and form a triangle congruent to ABC. What do you suppose might happen if you construct the Euler line of the triangle formed by those three Euler lines (if this gets too complicated you won’t miss much by skipping it)? Say ’maths is like that’ to form a conjecture.

This is called **Gossard’s theorem**. (If you google this it is advisable to remember that Gossard is a company that makes bras.)