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AFFORDANCES, CONSTRAINTS AND ATTUNEMENTS IN MATHEMATICAL ACTIVITY

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This paper explores one approach to describing learning mathematics as participation in practice. Greeno's articulation of affordances, constraints and attunements appears to provide a unified view which can be applied to mathematics learning at several levels. This framework can be useful in thinking about mathematical tasks and activity, although there are some characteristics of mathematical activity which are not well treated by it.

DEFINITIONS

The notion of 'affordances' arose in Gibson's work in 1950's as a way to grasp how learning takes place through perception of, and interaction with, an environment. It involves the common recognition that trying to deal with cognition separately from other factors, such as social settings, is unproductive. There needs to be a shift in focus, Greeno argues, from how individuals process information to an understanding of what information is available to use (1994). He (1998, p.9) sees affordances as "qualities of systems that can support interactions and therefore present possible interactions for an individual to participate in". Within systems there are norms, effects and relations which limit the wider possibilities of the system, that is constraints which are seen as "if-then relations between types of situations … including regularities of social practices and of interactions". Individuals acting with this system demonstrate attunements which are "regular patterns of an individual's participation … for example, well-coordinated patterns of participation in social practices". Constraints and affordances generate an 'ecology of participation'.

EXAMPLES

Mathematics lessons support a wide range of possible interactions such as: giving answers, talking about behaviour, discussing different ways of understanding a concept, social exchanges with peers, and so on. However, not all these possible kinds of interaction take place in all lessons because there are constraints operating. For example, one teacher may not permit social talk, so that indulging in social chat may result in exclusion; another teacher may always ask for explanations when an answer is given. There is much research in this area, some of it focusing on the systematic effects on particular groups.

Within a mathematics class, individuals express regular patterns of participation. Some will usually be first with their answers, others may never put their hands up but mutter to each other instead (Houssart, 2001). These attunements contribute to the practices of the classroom just as much as those emanating from the teacher, who also has regular patterns of participation, such as excluding noisy students, having a preference for offering a particular number of worked examples, and so on.

Although there is a lack of clarity about whether affordances and constraints are properties of the system or its participants' perceptions, such a framework can provide insight into the complexities of classrooms, schools as a whole, education systems and individual lessons. What can it offer which is specific to mathematics?

PROBLEMS

> Communities of practitioners share standards of what characterizes worthwhile problems to engage in and what constitutes an adequate or excellent solution of such a problem. (Greeno, 1998, p.10)

Standards are the constraints and affordances of a practice. Thus mathematics might be seen as a practice whose standards are expressed through the nature of the tasks with which mathematicians engage. However, this is not a very useful way to see school mathematics, since the tasks with which real mathematicians engage involve extended exploration, creation of new structures, argument, modelling, and no practice exercises! So to understand school mathematics as it is requires rejection of professional practice as a model and something else to be described in its place.

What are the worthwhile problems of school mathematics, and what constitute adequate solutions to such problems? Frequent testing and modularity have pushed school mathematics back towards seeing simple, one-stage problems with single answers as worthwhile, whereas the movement in the 80s was towards valuing multi-stage, ill-defined (mathematically) explorations for which the meaning of 'solution' was not clear. For many teachers, there is a conflict between beliefs about the value of mathematical thinking and the reduction of mathematics to learnt techniques for examinations. Instead of looking outside the classroom for what is seen as worthwhile, it makes much more sense to recognise that classrooms develop their own characterisations of worthwhile problems in which they will engage during mathematics lessons, and what is seen as an adequate solution is also very local.

A PARTICULAR TASK/ ACTIVITY

A task taken from the KS3 materials (DfEE, 2001, p.116) has been selected by the teacher, who has regular patterns of such choices, and is offered to students in the usual way for that class which is as something written on the board, with a title, "Pyramids", written above it.

$$3 \qquad\qquad x \qquad\qquad 5$$

A distinction between task and activity used by Christiansen and Walter (1986) is useful. The teacher has a *task* in mind but the students' *activity* is to begin to make their own sense which may or may not relate to the teacher's idea, and which is influenced by a variety of factors. There is a range of possible activities arising. Some people may think x is 4, others may want to add the three terms, others may be relating what they see to something seen before - is it a sequence of some kind, is order important? However, in this class the expected response is to wait for what the teacher does next so for many students the activity is to wait for more information. Inaction turns out to be a sensible choice. The teacher writes:

$$3 + x \qquad\qquad x + 5$$

Clearly this constrains what can happen next. Whether x is or is not 4 does not matter, but we are supposed to add adjacent terms, or at least to place a '+' sign between them, even if we cannot actually do the addition. The teacher has said "we are going to add this to this", but this adding does not produce an answer in an arithmetical sense. The next line is: $2x + 8$.

This is the template for what students do next. As a whole class they work on similar examples, several of them coming to the board to write up the next lines, or to offer their own examples, all ending in expressions of this linear type. Some of the examples have more than one x to start with; there is some discussion about how many starting positions could end up with the particular expression: $5x + 11$. The teacher sees class time as more usefully spent in discussing the development of complex examples and different ways to work with the structure; she gives practice examples for homework.

An analysis of the lesson as manifesting certain practices offers a way to see how the teacher and students together explore the mathematical situation, how the patterns of participation develop as students took part, or chose not to take part, and how they were encouraged to work, and how and why they were prepared to engage with the task. This could be achieved through discourse or interaction analysis, but we could also analyse it by looking for affordances and constraints of the task. The possibilities offered by the task, the teacher, the students and the classroom are constrained by the interactional norms they have developed, the examples which the teacher and students offer during the lesson, and the regularities of individual participation. These would give a rich account of the lesson. This lesson could be seen as a good lesson by many standards, such as those of inspection, or of qualified teacher status, or of those considering inclusion.

But something is missing from such an analysis – the relationships between the activity in that classroom and mathematics as a wider practice. A mathematician who is not a participant in that classroom might ask "what is this structure? how can we end up with more x's than we started with?". Indeed, some of the students may have been thinking that as well, but as a participant in that lesson they put that question aside and accept the norms of the task - their question was inappropriate in the context of 'pyramids'. Not asking such questions, and not hypothesising about the first thing the teacher writes, are constraints.

Suppose we analyse the task in terms of affordances, constraints and attunements. The possibilities offered by the first line in terms of supporting interactions have already been discussed, but these are rapidly constrained by the next line. In mathematics structures are displayed which inform us about relationships. From the second line we learn more about the first. Students become attuned to this, and know they can either wait for the second line or that when it appears they have to channel their previous sense-making in accordance with it. Faced with a choice between making sense early, which may then have to be altered, or waiting until there is more information, the temptation to remain passive until all is revealed might be quite strong. Indeed, in this task it would be positively unhelpful to bring to bear anything already known about mathematics on the first line, for these pyramids have a logic and purpose which does not relate easily to other mathematical structures.

ANOTHER TASK/ACTIVITY

Suppose instead that what the teacher had in mind is something else, and what is written on the board is:

$$3 \qquad\qquad x \qquad\qquad 5$$

with no title. She then asks: “what could *x* be?” thus indicating that this is what students must think about, they cannot sit and wait for something else to happen. They have to make personal sense of what is written, and there are no more clues apart from the fact that this is a mathematics lesson, so normal behaviour for such lessons and normal mathematical procedures and knowledge are available to be used. Obviously sequences and order come to mind, and possibly an assumption that there is enough information there to give a value for *x*, which seems to be physically mid-way between 3 and 5. What is written affords a range of mathematical responses, each of which arises from some activity. The question constrains students to think about a value for *x*; the normal classroom practice always allows a range of conjectures (including guessing what the teacher is thinking). She offers:

$$3 \quad x \qquad\qquad\qquad 5$$

“What could *x* be now?” This new question constrains what students can think about. They are being directed, by position, to imagine that the position of *x* might be important, or to examine whether their original idea depends on the position of *x*. A student who was thinking about ‘between-ness’ might be comfortable, but one who was thinking about ‘the mean’ might be jolted a little. It is extremely unlikely that anyone will be thinking “oh look, if I add these in pairs I will eventually get $2x + 8$”. The action of introducing a new example is a constraint on affordances, and constraints are not inherently negative. Much mathematics can be seen as constraints on freedom: a straight line involves constraining the relationship between two variables; the real numbers involve a constraint on the complex plane.

What the student sees, feels allowed to see, or is able to articulate, depends on regular patterns of engagement with mathematics as well as the affordances and constraints. We can, therefore, learn a lot about students’ responses to tasks by thinking about what the task affords in terms of activity, what is constrained and what attunements are brought to bear in the activity. Although the practices in the two classrooms could be very similar in terms of participation, involvement, and many other superficial, observable social features, the mathematical practices are rather different. In the first, students wait until the teacher does something unexpected, and mathematics does not need to make connected sense across lessons so long as the social practices of the classroom keep everyone on board. In the second, students need to connect mathematics across lessons, and even across teachers, classrooms and schools, in order to participate. There is a sense in which they are operating within mathematical practices which overarch particular classrooms and teachers; generalisable mathematical practices.

INDIVIDUAL LEARNING

Greeno,and other socio-cultural theorists, see learning as improved participation in interactive systems – becoming better attuned to constraints and affordances of activity systems. As a description of how we come to know everything we know, including how to behave in mathematics classrooms and when faced with mathematical situations, this functions well, even on the micro-level of looking at mathematical tasks and activity. However, this only goes so far as we can extend our understanding of 'interactive' what I experience myself when working alone with a textbook. Greeno also recognises this when, with Boaler, he says:

> The conceptual framework we have used … would need to be extended to accommodate examples of engaged conceptual knowing that is only weakly supported by discourse interactions in the individual's immediate learning community…. It could involve hypothesizing a form of connected knowledge that emphasises the knower's being connected with the contents of a subject-matter domain**.** (Boaler and Greeno, 2000, p.191)

This acknowledges a weaknesses of the socio-cultural project which, while being valuable at many levels of mathematics education, provides an inadequate framework for looking at how "the knower might be connected with the subject-matter domain".

The analysis of the lessons above indicates that a more fruitful approach might be to focus on the affordances and constraints of the task in terms of variation and the previous experience of learners which they bring to bear on it. Variation and experience can provide a rich analysis of response to certain kinds of task.

For example, consider this secondary task:

Write down a quadratic whose roots differ by 2.
Write down another quadratic whose roots differ by 2.
Write down another quadratic whose roots differ by 2.

Responses to these types of task have been discussed in depth elsewhere (Watson and Mason, 2004). Learners have to make decisions about whether and how to vary their answers, and these responses reveal the knowledge of patterns and possibilities which have occurred to them. Rather than seeing their answers as chosen from some hegemonistic space of possible answers (*the* quadratics or *the* numberline) it is more useful to see them as emanating from a personal, situated example space which has arisen through the interaction between their experience, their reading of the task, the environment and so on. Of course there are social aspects to the activity, such as guessing what the teacher will do with their answers, but they are still selecting from their own spaces as a starting point.

Sharing such responses can offer possibilities for extending example spaces, either temporarily for that lesson or so that richer spaces come to mind in future lessons. Thus, if example spaces are individual, situationally and temporarily specific, triggered into use by the task and environment we can see learning as

> Extending the contents and connections in the current example space, and extending and enhancing what spaces might be available in future situations, through interaction

This comes very close to Marton's definition of learning as discernment of variation in near-simultaneously occurring events (Marton, 2001), which does not deal with *how* one discerns, but *what* one discerns. In the analysis of lessons above I have gone further and dealt with what is available to be discerned. Thus I have made the shift discussed by Greeno from process to what is available to be processed.

If learning is "improved participation in interactive systems – becoming better attuned to constraints and affordances of activity" then to understand the learning of conventional school mathematics we need to look in detail at the constraints and affordances of mathematical sense-making, so closely in fact that we are right up against mathematical variation, mathematical constraint, and mathematical generalisation.

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