Repressed images

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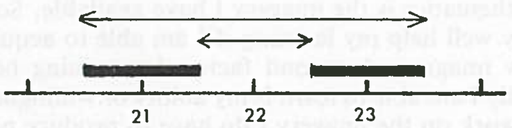
I was talking to some teachers about mental images and tried to find out if they carried number lines in their heads. One teacher was sure she did not. Later I asked them to work out something like $69+25$ mentally. The person in question explained how she did it: 69, 79, 89, 94. At the same time her right hand indicated jumps to the right, two large followed by one small. I pointed this out to her and she took a while to recognise what she had done, and that her hand movements were related to some sort of number line.

Later, I was teaching some students and we were discussing errors. In particular there was the problem of the relatively large error that is likely to occur when subtracting two nearly equal quantities for each of which there is some error. We had looked at some examples like

$$23 \pm 0.5 - 21 \pm 0.5 = 2 \pm 1$$

and somehow it was getting a bit laboured. I suddenly realised that the situation was very clear to me because I had a mental picture of what was going on:



but for some curious reason I was not sharing this with the students. I drew it on the board and some said yes, that made it all clear.

I find this puzzling. I am very aware of the large extent to which I use images of one sort or another in mathematics, and from talking to other people, know that at least some people also find them valuable. I am also very keen on using diagrams whenever possible in mathematics, and encourage students to do so too. But here was a situation where I use an image and find it helpful, but where it didn't readily come out in my trying to communicate what I was thinking. Despite my interest in visual representation and my conviction of its importance, I didn't immediately resort to it.

What then of the teacher? My interpretation is that she has some sort of number-line image which guides some of her thinking about numbers. But she is still less aware of it than I am of mine, and so, perhaps, is less likely to try to develop such a useful image in her children.

There seem to me to be several things to be said about this. One is about the nature of knowledge. So much knowledge, in other areas as in mathematics, is taken to exist in statements: Paris is the capital of France, $69+25=94$. The statement appears to encapsulate all that needs to be said on the subject — and therefore making the statement is a satisfactory way of confirming that you have the knowledge. A picture or a diagram does not make a statement of this sort, so its status as an expression of knowledge is at best uncertain and, in the case of a lot of mathematics writing, downright suspect. Such suspicions may be part of the reason why diagrams are *de trop*.

Another point is about the use of gestures. I have taken to watching people's hands when they talk about mathematics. It's very interesting because they seem sometimes to describe ways of thinking which the speaker neither expresses in his words, nor shows any desire to communicate by drawing. The gestures, moreover, are not designed really to communicate to the listener: they seem to be part of the speaker's managing his own thoughts. (Next time you ask someone to direct you to a strange place, watch his hands).

The use of gestures makes me wonder if 'images' is not a misleading term. It suggests internalised visual representation, and sometimes I think it is more like internalised body movements, internalised action. If this is anything like the case, for at least some people, then perhaps a useful teaching approach might sometimes be to get learners to carry out the appropriate movements. Infants walk along number steps, juniors move their fingers along counting strips, my students now sketch graphs in the air (they seem to quite like it).

Then there is the relationship between knowledge and communicating. Maybe knowledge does exist in statements (though I find this difficult to understand), but, even so, knowledge certainly is not communicated by making statements. Otherwise we could tell children their tables and go home. Nevertheless, a lot of the time we act as though this were the case. But when we communicate with ourselves we don't bother much with statements. On paper we make sketches, diagrams, rough workings, seldom attending to grammatical construction of what we write. In our heads I don't think we use statements very much — vaguely, we manipulate ideas and images. And, perhaps, we talk to ourselves with gestures. Why then when we come to communicate our ideas to others are we so concerned with statements? Why don't we share the diagrams, models, metaphors that buzz around in our heads? Why do we repress our images?

The above article was passed round to various people for further comment and discussion. Two replies, from Derek Ball and Dick Tahta, are given below, with a final word from Stuart Plunkett. Readers are invited to continue the discussion.

Comment 1

I, also, am not very clear about what it means to know some mathematics. My mathematical knowledge certainly does not seem to me to consist of statements; perhaps it consists of ideas and the statements are attempts to make the ideas public. There are a number of different statements I can make in an attempt to communicate an idea. If I make a statement to you, then you may be catching on to my idea — more or less; on the other hand, my statement may be quite meaningless to you, or, perhaps even more disconcertingly, it may suggest to you some completely different idea. I do not find it easy to say what I mean by an idea or to explain what relation it has to an image. I think it helps me if I think of images as internalised experiences of some kind (some of which will be imagined experiences); in this sense, I can understand the phrase 'internalised action'. Images do not have to be visual; they may be associated with any or all of the senses. Whereas images are haphazard, I have to do some work to get an idea. Images just come, or else I generate them involuntarily while working on some idea. I am sure that I need images to make sense of my ideas. The relating of images to ideas can happen in both directions; I may be aware of having some images and find myself working on them generating ideas; on the other hand, someone else may have made a public statement or asked a question (for example, what is $69+25$?) and I may have some idea about it, in which case I may find myself looking for an image (I already have) to help with the idea.

I am not sure what it means to repress images. I may have certain images at my disposal (and it may be hampering me that my imagery is not richer or more extensive). I may be in the habit of working on ideas and inevitably using what imagery I have. I may find myself making public statements in an attempt to communicate my ideas. I may believe that these statements are the only way of communicating the ideas and that anyone who hears the statements will certainly be able to share the ideas. On the other hand, if I try to communicate the imagery I have, which helps me to make sense of an idea, then I must first do some work on that imagery and have some ideas about it. Then, when I try to share my imagery, what I am really sharing is my ideas about my imagery. The sharing of these ideas seems as likely to be problematic as the sharing of any other ideas. If I talk to you about addition you may well find it helpful if I try to draw your attention to a number line (though conceivably you will not). But if I do talk to you about a number line, I am not sure that I would want to think of this as sharing my imagery, but rather that I have done some work on my ideas about addition, as a result of which I am trying to communicate somewhat different ideas, which may (or perhaps may not) be more helpful. Nevertheless, it is worthwhile for me to consider whether some of the ideas I try to communicate are the poorer for not having been worked on enough and that the richness which may come from adding diagrams or spatial or other aspects to my ideas may well be helpful to you if you are trying to understand my ideas.

It seems to me that one of the factors which determine how easily I am able to learn some mathematics is the imagery I have available. So it may well help my learning if I am able to acquire new imagery. A second factor determining how easily I am able to learn is my ability or willingness to work on the imagery I do have to produce new ideas. So perhaps the main task of the teacher is seduction. The teacher needs to seduce the learner into being open to experiences of certain kinds, so that he is likely to acquire new imagery; the teacher also needs to seduce the learner into wanting to work on the imagery he does have to produce new ideas. Putting it another way, the teacher needs to give the learner confidence and also to make the learner excited about mathematics. After that the learning must be left to the learner.

D.B.

Comment 2

The issue is not, I suppose, about images that are *repressed*, for these are necessarily not known to the conscious self, but rather about images that are, for some reason or other, *withheld*. There may indeed be excellent reasons for such withholding and, in general, I often wish people were more reserved about their private images. I do not always want to know other people's dreams and sometimes personal images are promiscuously spread abroad without restraint or, as with some makers of films, without a sign of artistically fashioned form.

The person who always sees the number seven as a dagger dripping blood may be having a vivid and important experience, but it is not necessarily one that can be usefully shared except for individual therapeutic purposes. A psychoanalyst reports that a patient always wanted to be able to do mathematics, but was unable to manipulate the symbols confidently enough. "Mathematics has always had great emotional significance for me as something I can't do as well as I'd like to," said the patient. "The integral sign has always stood for the unapproachable or forbidden." These may be extreme examples, but they serve to remind us that there is some sense in the tradition that seeks to avoid explicit promulgation of private images in mathematics in order to leave people free to summon and operate on their own. In our horror at some of the sterile aspects of this tradition it is worth remembering that there is *some* point in, for example, printing geometry books without diagrams, daft as this sometimes seems.

It is not difficult to tap the world of private imagery. Given some trust in the classroom and familiarity with such work, children can and will report vivid images of apparently neutral mathematical elements. The following extracts from some writings by nine and ten-year-olds about their 'inner pictures' speak for themselves.

"My point was the tip of a triangle . . . my triangle started off in complete darkness, then suddenly it seemed to be a kind of lawn with a path on it . . . It was funny, but I seemed to be the triangle and yet I saw it with my own eyes."

"The big circle is a big strong energetic man. The small circles represent clothes that have shrunk after being washed. The man, of course, can't get into them."

Jung claimed that when geometrical elements appear in dreams or drawings they are original images of what he calls the primeval condition. He emphasised that great care and sensitivity were required in dealing with these matters. ". . . the tongue and the hand are, of course, possessed by something other than oneself . . . *probate spiritus* — watch which spirits are protecting you . . . it's as if such voices were autonomous."

I think it is this sense of an autonomous presence that makes the attempted sharing of images in mathematics so powerful and so important; and, incidentally, which gives mathematics its sense of an objective existence. Clearly we do a disservice to students, as Stuart Plunkett points out, if we hide from them the fact that we do operate on images while we murmur words. We certainly need to encourage people to work on *their* internalised images. The crucial pedagogical question seems to be whether there are some images that are, in some senses, so universal and so over-whelming that they need to be suggested — offered — in some way. Generations of teachers and text-book writers have certainly assumed that this is the case and students of all ages are offered a host of images — number-lines, pie-charts, graphs, abacus-spikes and so on.

What seems to be missing is any criterion for choosing worthwhile images. Are some that are usually offered just not good enough, indeed perhaps harmful in some way? The pie-chart for fractions would be my example. Are some outstandingly multivalent and somehow inevitable? The square lattice, on geoboard or paper, would be, for me, one of these. Are there some images that are too sophisticated for beginners and so demanding *substitutes?* My example would be complementing-in-ten through folding and unfolding fingers, and calculus teaching offers many more.

D.T.

Comment 3

It may be, as Dick says, that it is not difficult to tap the world of private imagery. It may be that it is not difficult to get learners to work on their images. And I agree with Derek that sharing images is as problematic as any other form of communication. The amazing fact remains that, by and large, we do *not* work with learners on their imagery. No, I am not proposing the imposition of a set of images on reluctant learners. What I am eager to point out is that in doing mathematics imagery seems to be pretty important, and that this fact goes largely unacknowledged in mathematics teaching. Students may well be offered some sort of image but they are rarely invited to work on this. For most learners mathematics is the passive listening to words and the algorithmic manipulation of symbols. While we hunt for the universal images let us at least spread the news that mathematics can be represented and worked on in spatial, diagrammatic, kinaesthetic forms.

S.P.