

\*31. Consider the following 16 assertions, where  $\exists$  means "there exists,"  $\forall$  means "for any" and  $\ni$  means "such that":

- 1)  $\exists \varepsilon > 0 \exists k \exists n > k \ni |x_n - a| < \varepsilon$ ;
- 2)  $\exists \varepsilon > 0 \exists k \exists n > k \ni |x_n - a| \geq \varepsilon$ ;
- 3)  $\exists \varepsilon > 0 \exists k \forall n > k \ni |x_n - a| < \varepsilon$ ;
- 4)  $\exists \varepsilon > 0 \exists k \forall n > k \ni |x_n - a| \geq \varepsilon$ ;
- 5)  $\exists \varepsilon > 0 \forall k \exists n > k \ni |x_n - a| < \varepsilon$ ;
- 6)  $\exists \varepsilon > 0 \forall k \exists n > k \ni |x_n - a| \geq \varepsilon$ ;
- 7)  $\exists \varepsilon > 0 \forall k \forall n > k \ni |x_n - a| < \varepsilon$ ;
- 8)  $\exists \varepsilon > 0 \forall k \forall n > k \ni |x_n - a| \geq \varepsilon$ ;
- 9)  $\forall \varepsilon > 0 \exists k \exists n > k \ni |x_n - a| < \varepsilon$ ;
- 10)  $\forall \varepsilon > 0 \exists k \exists n > k \ni |x_n - a| \geq \varepsilon$ ;
- 11)  $\forall \varepsilon > 0 \exists k \forall n > k \ni |x_n - a| < \varepsilon$ ;
- 12)  $\forall \varepsilon > 0 \exists k \forall n > k \ni |x_n - a| \geq \varepsilon$ ;
- 13)  $\forall \varepsilon > 0 \forall k \exists n > k \ni |x_n - a| < \varepsilon$ ;
- 14)  $\forall \varepsilon > 0 \forall k \exists n > k \ni |x_n - a| \geq \varepsilon$ ;
- 15)  $\forall \varepsilon > 0 \forall k \forall n > k \ni |x_n - a| < \varepsilon$ ;
- 16)  $\forall \varepsilon > 0 \forall k \forall n > k \ni |x_n - a| \geq \varepsilon$ .

Interpret these assertions in terms of familiar properties of sequences or their negatives.

*Answer.* 1) Every sequence  $\{x_n\}$  has this property; 2) Not every term of  $\{x_n\}$  equals  $a$ ; 3) The sequence  $\{x_n\}$  is bounded; 4) The point  $a$  is not a limit point of  $\{x_n\}$ ; 5) The sequence  $\{x_n\}$  does not approach infinity; 6) The number  $a$  is not the limit of  $\{x_n\}$ ;